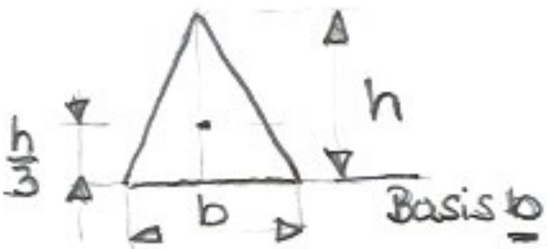
Beispiele zu Trägheits- u. Widerstandsmomenten



Bsp 1: Bestimmung des reduzierten FTM in Bezug auf die Basis b.

allg.: $I_b = I + k^2 \cdot A$

$$I_b = \frac{b \cdot h^3}{36} + \left(\frac{h}{3}\right)^2 \cdot \frac{b \cdot h}{2}$$

$$I_b = \frac{b \cdot h^3}{36} + \frac{h^2}{9} \cdot \frac{b \cdot h}{2}$$

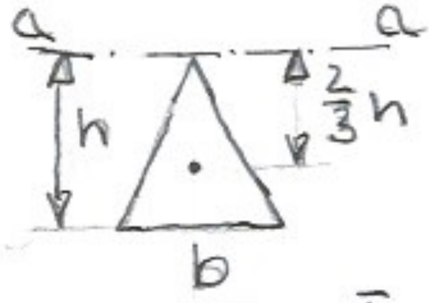
$$I_b = \frac{b \cdot h^3}{36} + \frac{b \cdot h^3}{18} = \frac{b \cdot h^3 + 2b \cdot h^3}{36}$$

$$\underline{\underline{I_b = \frac{1}{12} b \cdot h^3}}$$

$A_\Delta = \frac{b \cdot h}{2}$

$k = \frac{h}{3}$

Bsp. 2: Bestimmung des reduzierten Trägheitsmomentes eines Dreiecks auf eine Achse durch die Spitze



allg.: $I_a = I + k^2 \cdot A$

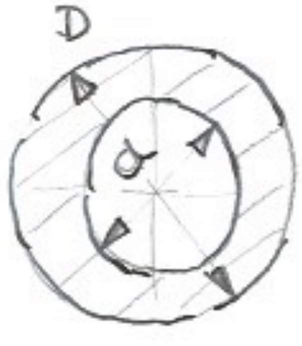
$$I = \frac{b \cdot h^3}{36} \; ; \; k = \frac{2}{3} h \; ; \; A = \frac{1}{2} h \cdot b$$

$$I = \frac{b \cdot h^3}{36} + \frac{4}{9} h^2 \cdot \frac{1}{2} h \cdot b$$

$$I = \frac{b \cdot h^3}{36} + \frac{4}{18} b \cdot h^3 = \frac{b \cdot h^3 + 8 \cdot b \cdot h^3}{36}$$

$$\underline{\underline{I = \frac{1}{4} b \cdot h^3}}$$

Bsp. 3: Das äquatoriale Trägheitsmoment für einen Kreisring ist zu bestimmen.



$$I_{ges} = I_{D} - I_{d}$$

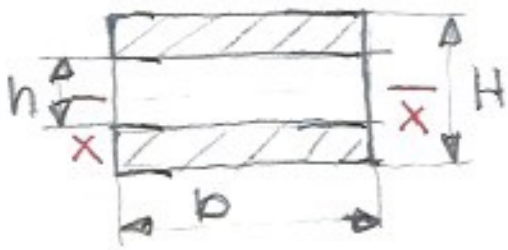
$$= \frac{\pi}{64} D^4 - \frac{\pi}{64} d^4$$

$$I = \frac{\pi}{64} \left(D^4 - d^4 \right)$$

Widerstandsmoment:

$$W = \frac{I}{e} = \frac{\pi}{64} \frac{(D^4 - d^4)}{D}$$

Bsp.: 4 Bestimmen Sie das äquatoriale Trägheitsmoment für nachfolgende Skizze



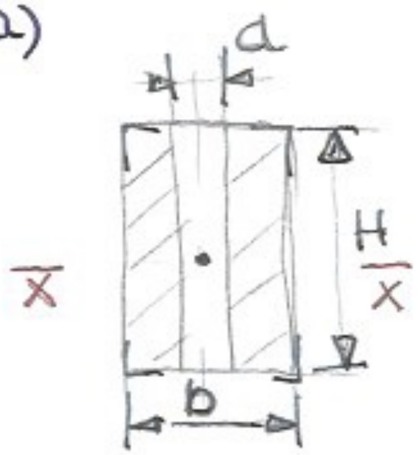
$$I_{ges} = \frac{b \cdot H^3}{12} - \frac{b \cdot h^3}{12}$$

$$I_{ges} = I_x = \frac{b}{12}\left(H^3 - h^3\right)$$

$$W_x = \frac{I_x}{e} = \frac{\frac{b}{12}\left(H^3 - h^3\right) \cdot 2}{H} = \frac{b}{6H}\left(H^3 - h^3\right)$$

Bsp.: 5 Verschiedene Querschnittsformen I_x, W_x bestimmen

a)



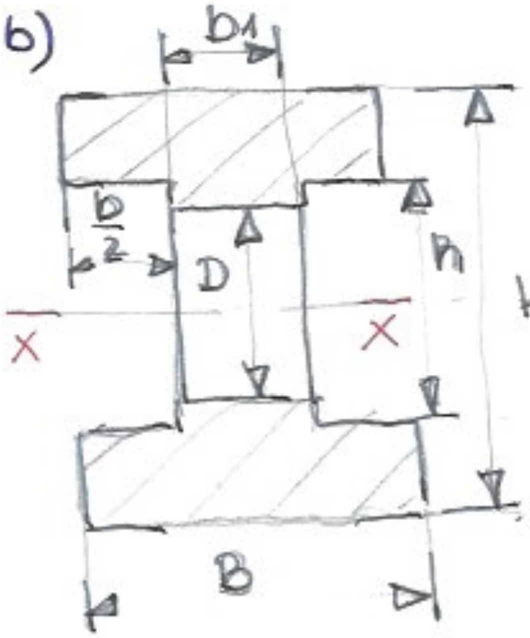
$$I_x = \frac{b \cdot H^3}{12} - \frac{a \cdot H^3}{12}$$

$$I_x = \frac{H^3}{12}(b - a)$$

$$W_x = \frac{\frac{H^3}{12}(b-a) \, 2}{H}$$

$$W_x = \frac{H^2}{6}(b-a)$$

b)



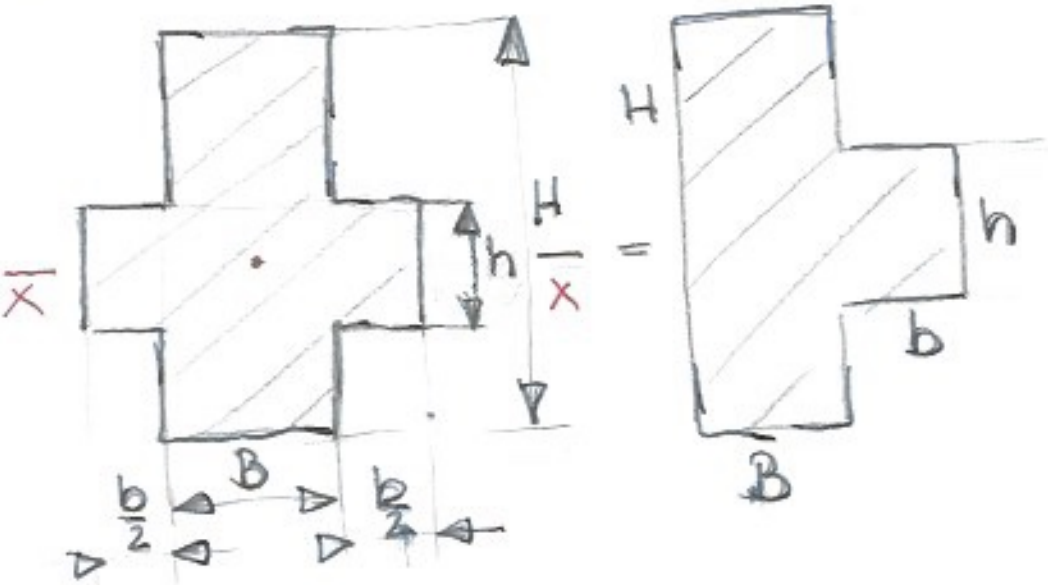
$$I_x = \frac{B\cdot H^3}{12} - \frac{b_1\cdot D^3}{12} - \frac{b\cdot h^3}{12}$$

$$I_x = \frac{1}{12}\left(B\cdot H^3 - b\cdot h^3 - b_1\cdot D^3\right)$$

$$W_x = \frac{1}{12}\cdot\frac{2}{H}\left(B\cdot H^3 - b h^3 - b_1 D^3\right)$$

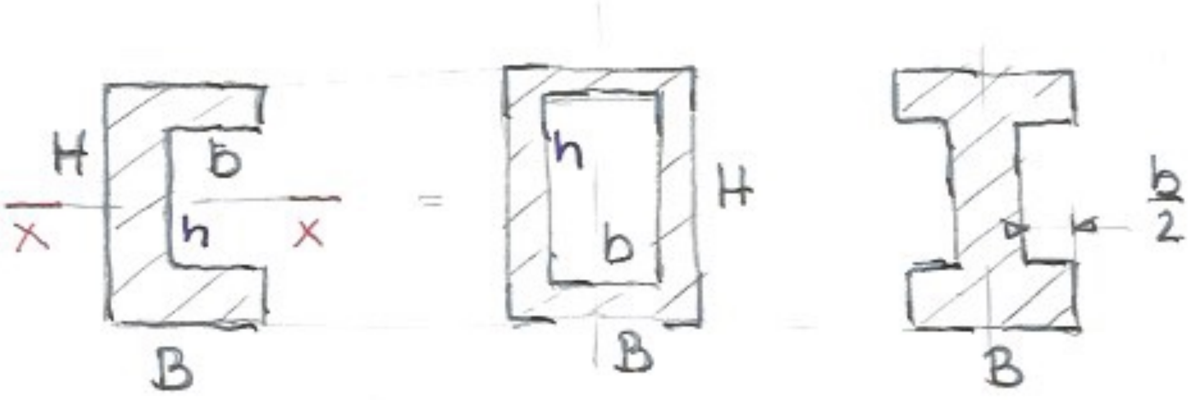
$$W_x = \frac{1}{6H}\left(B\cdot H^3 - b\cdot h^3 - b_1\cdot D^3\right)$$

c)



$$I_x = \frac{B\cdot H^3}{12} + \frac{b\cdot h^3}{12} = \frac{1}{12}\left(B\cdot H^3 + b\cdot h^3\right)$$

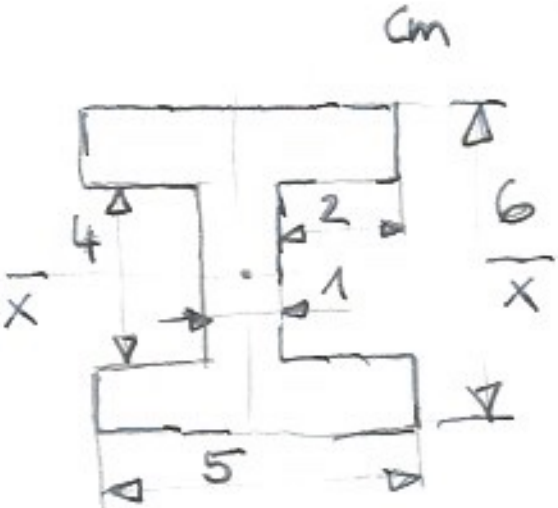
$$W_x = \frac{1}{6H}\left(B\cdot H^3 + b\cdot h^3\right)$$



$$I_x = \frac{B \cdot H^3}{12} - \frac{b \cdot h^3}{12} = \frac{1}{12}\left(B \cdot H^3 - b \cdot h^3\right)$$

$$W_x = \frac{1}{6H}\left(B \cdot H^3 - b \cdot h^3\right)$$

Zahlenbeispiel:



$$I_x = \frac{5 \cdot 6^3}{12} - \frac{4 \cdot 4^3}{12}$$

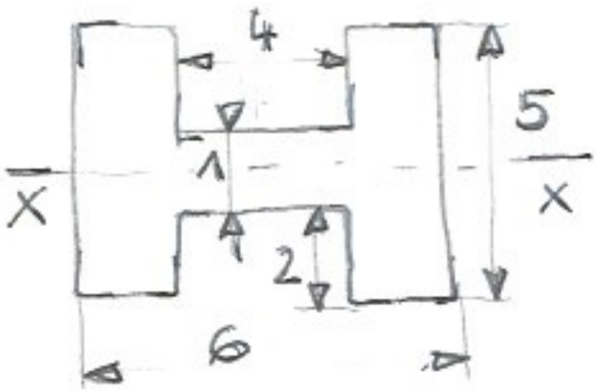
$$I_x = \frac{1}{12}\left(5 \cdot 6^3 - 4 \cdot 4^3\right)$$

$$\underline{\underline{I_x = 68{,}6 \text{ cm}^4}}$$

$$W_x = \frac{I_x}{e} = \frac{68{,}6 \text{ cm}^4}{3 \text{ cm}}$$

$$\underline{\underline{W_x = 22{,}86 \text{ cm}^3}}$$

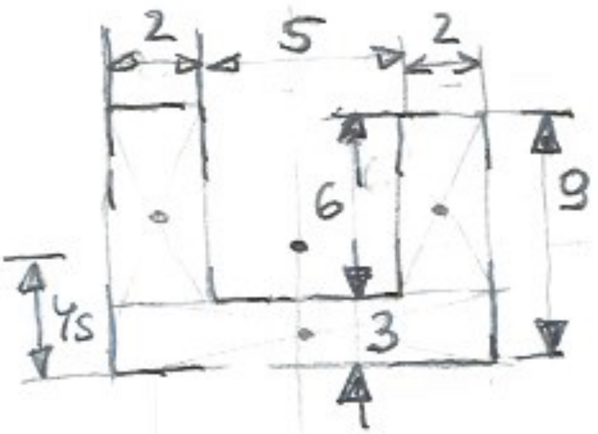
cm



$$I_x = \frac{2 \cdot 5^3}{12} + \frac{4 \cdot 1^3}{12}$$

$$\underline{\underline{I_x = 21{,}16\ cm^4}}$$

$$W_x = \frac{I_x}{e} = \underline{\underline{8{,}46\ cm^3}}$$



i	A_i	y_i	$A_i y_i$
1	$3 \cdot 9 = 27$	1,5	40,5
2	$2 \cdot 6 = 12$	6	72
3	$2 \cdot 6 = 12$	6	72
	51		184,5

$$y_s = \frac{184{,}5}{51} = 3{,}617\ cm$$

i	I_{xi} (cm⁴)	y_i	A_i	$A_i \cdot y_i^2$ (cm⁴)
1	$\frac{9 \cdot 3^3}{12} = 20{,}25$	$3{,}6 - 1{,}5 = 2{,}1$	27	119,07
2	$2\left(\frac{2 \cdot 6^3}{12}\right) = 78{,}54$	$6 - 3{,}6 = 2{,}4$	12	$2 \cdot (69{,}12) = 138{,}24$
	98,79			257,3

$$I_x = 92{,}25 + 257{,}3 = 356{,}1\ cm^4$$

bezüglich UK (andere Einteilung)

$$I_u = \frac{1}{3}\left(2\cdot 9^3 + 2\cdot 9^3 + 5\cdot 3^3\right)$$

$$I_u = 1017\ cm^4$$

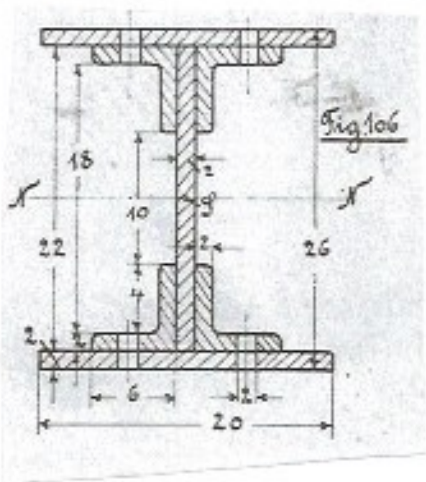
$$I_x = I_u - y_s^2 \cdot A_{ges}$$

$$I_x = 1017 - 3{,}6^2 \cdot 51$$

$$\underline{\underline{I_x = 356\ cm^4}}$$

Bsp.:

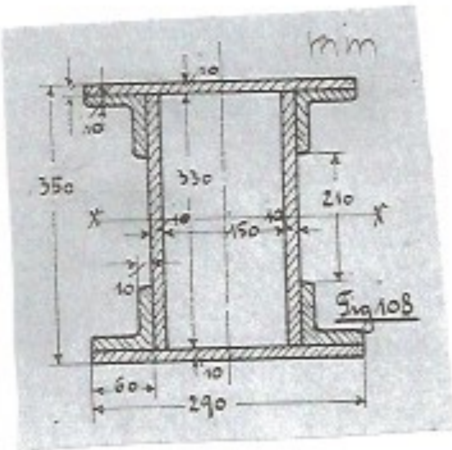
cm



$$I_N = \frac{(20-4)\cdot 26^3}{12} - \frac{6\cdot 22^3}{12} - \frac{4\cdot 18^3}{12} - \frac{4\cdot 10^3}{12}$$

$$I_N = 15833{,}3\ cm^4$$

$$W_N = 1218\ cm^3$$



$$I_x = \frac{290\cdot 350^3}{12} - \frac{150\cdot 330^3}{12} - \frac{20\cdot 210^3}{12} - \frac{100\cdot 310^3}{12}$$

$$I_x = 32\,324\ cm^4$$

$$W_x = 1847\ cm^3$$