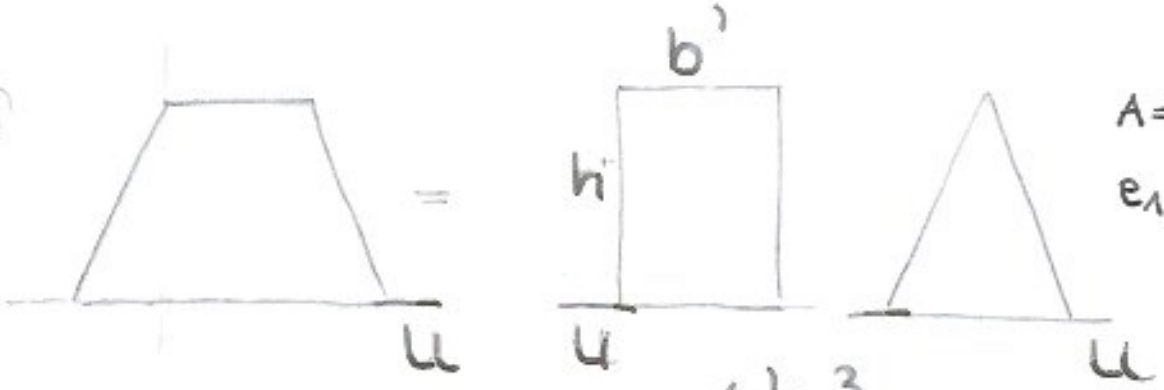
I_x - bestimmen



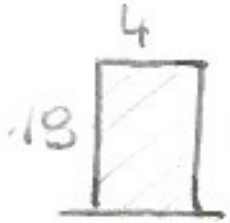
$A = \frac{a+b}{2} \cdot h$

$e_1 = \frac{h}{3} \cdot \frac{a+2b}{a+b}$

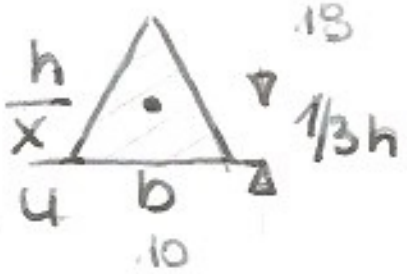
$I_u = \frac{1}{3} b' \cdot h^3$

$A = 162\ cm^2$

$e_1 = 7{,}33\ cm$



$I_u = \frac{4 \cdot 18^3}{3}$



$I_x = \frac{b \cdot h^3}{36}$; $A = \frac{b \cdot h}{2}$; $k = \frac{1}{3} h$

$I_u = I_x + k^2 \cdot A$

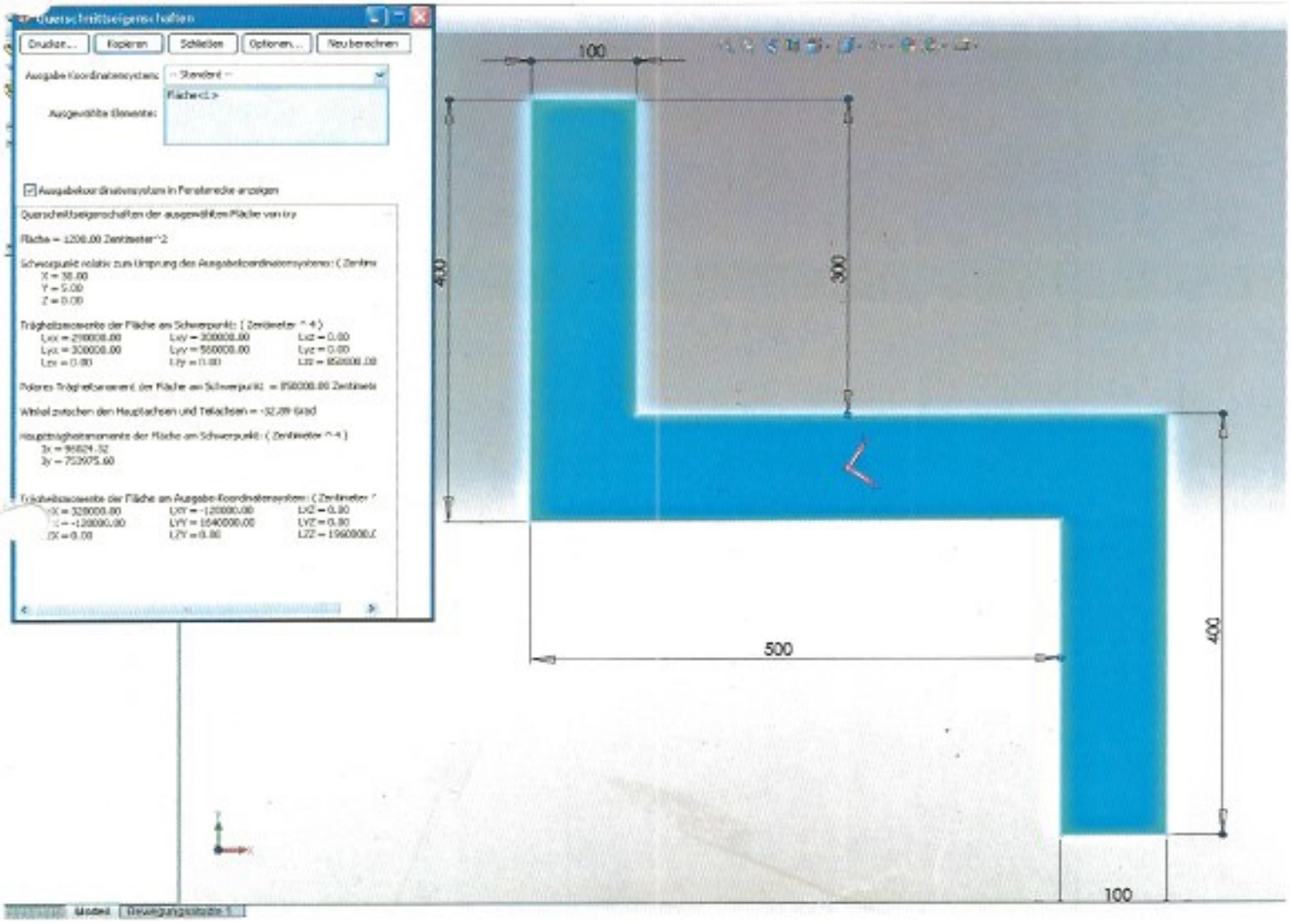
$I_u = \frac{b \cdot h^3}{36} + \frac{1}{9} h^2 \cdot \frac{b\,h}{2} = \frac{3}{36} b h^3$

$I_u = \frac{1}{12} b \cdot h^3$

$I_{x\triangle} = \left(\frac{b' \cdot h^3}{3} + \frac{b \cdot h^3}{12} \right) - e_1^2 \cdot A_{\triangle}$

$I_{x\triangle} = \left(\frac{4 \cdot 18^3}{3} + \frac{10 \cdot 18^3}{12} \right) - 7{,}33^2 \cdot 162$

$I_{x\triangle} = \underline{3931{,}9\ cm^4}$



Querschnittseigenschaften der ausgewählten Fläche von ixy

Fläche = 1200.00 Zentimeter^2

Schwerpunkt relativ zum Ursprung des Ausgabekoordinatensystems: (Zentimeter)
X = 30.00
Y = 5.00
Z = 0.00

Trägheitsmomente der Fläche am Schwerpunkt: (Zentimeter ^ 4)

Lxx = 290000.00	Lxy = 300000.00	Lxz = 0.00
Lyx = 300000.00	Lyy = 560000.00	Lyz = 0.00
Lzx = 0.00	Lzy = 0.00	Lzz = 850000.00

Polares Trägheitsmoment der Fläche am Schwerpunkt = 850000.00 Zentimeter ^ 4

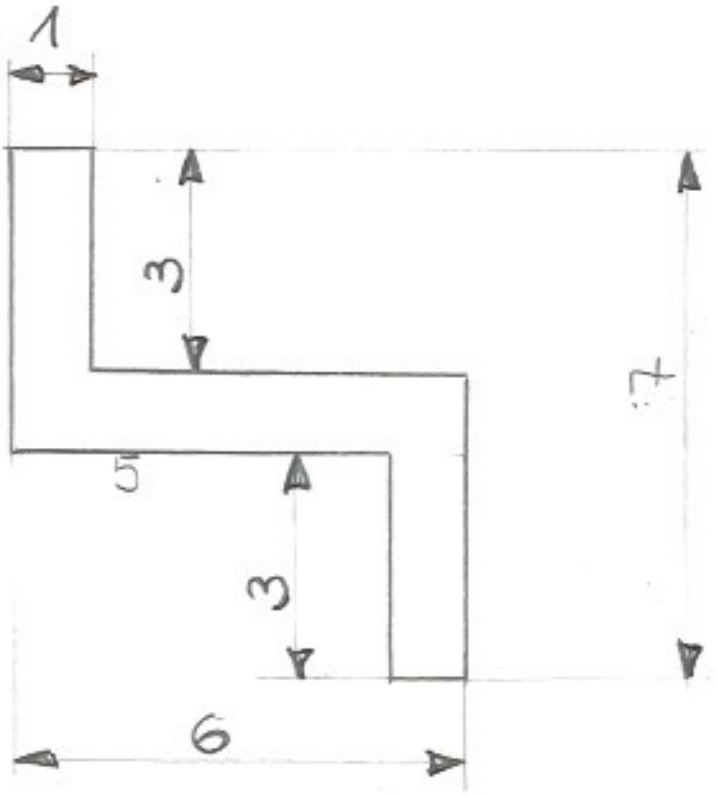
Winkel zwischen den Hauptachsen und Teilachsen = -32.89 Grad

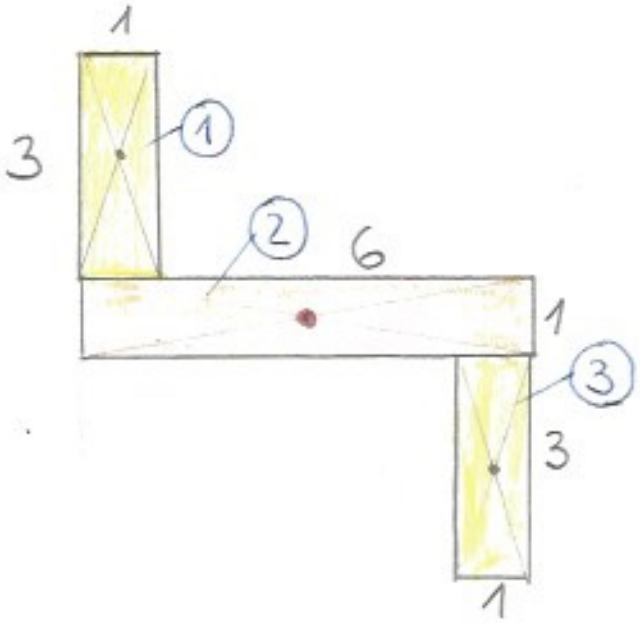
Hauptträgheitsmomente der Fläche am Schwerpunkt: (Zentimeter ^ 4)
Ix = 96024.32
Iy = 753975.68

Trägheitsmomente der Fläche am Ausgabe-Koordinatensystem: (Zentimeter ^ 4)

LXX = 320000.00	LXY = -120000.00	LXZ = 0.00
LYX = -120000.00	LYY = 1640000.00	LYZ = 0.00
LZX = 0.00	LZY = 0.00	LZZ = 1960000.00

[dm]





ges.: I_x; I_y; I_{xy}; I_{max}; I_{min}; φ

I_x

λ	I_{xi}	A_i	Δy_i	$A_i k_i^2$
1	$\frac{1 \cdot 3^3}{12} = 2{,}25$	3	2	12
2	$\frac{6 \cdot 1^3}{12} = 0{,}5$	6	0	0
3	$\frac{1 \cdot 3^3}{12} = 2{,}25$	3	-2	12
	Σ 5			24

$I_x = 29\ dm^4$

I_y

			Δx	
i	I_{yi}	A_i	k_i	$A_i \cdot k_i^2$
1	$\frac{3 \cdot 1^3}{12} = 0{,}25$	3	-2,5	18,75
2	$\frac{1 \cdot 6^3}{12} = 18$	6	∅	∅
3	$\frac{3 \cdot 1^3}{12} = 0{,}25$	3	2,5	18,75
Σ	18,5			Σ 37,5

$I_y = 56\ dm^4$

I_{xy}

λ	$I_{x'y'}$	Ai	dx	dy	Ai · dx · dy
1	∅	3	-2,5	2	-15
2	∅	6	∅	∅	∅
3	∅	3	2,5	-2	-15
				Σ	-30

$\underline{I_{xy} = 30\ dm^4}$ $\qquad I_{xy} = -\left[\sum_{i=1}^{n} Ai \cdot \Delta x \Delta y\right]$

Zusammenfassung

$I_x = 28\ dm^4$
$I_y = 56\ dm^4$
$I_{xy} = 30\ dm^4$

$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

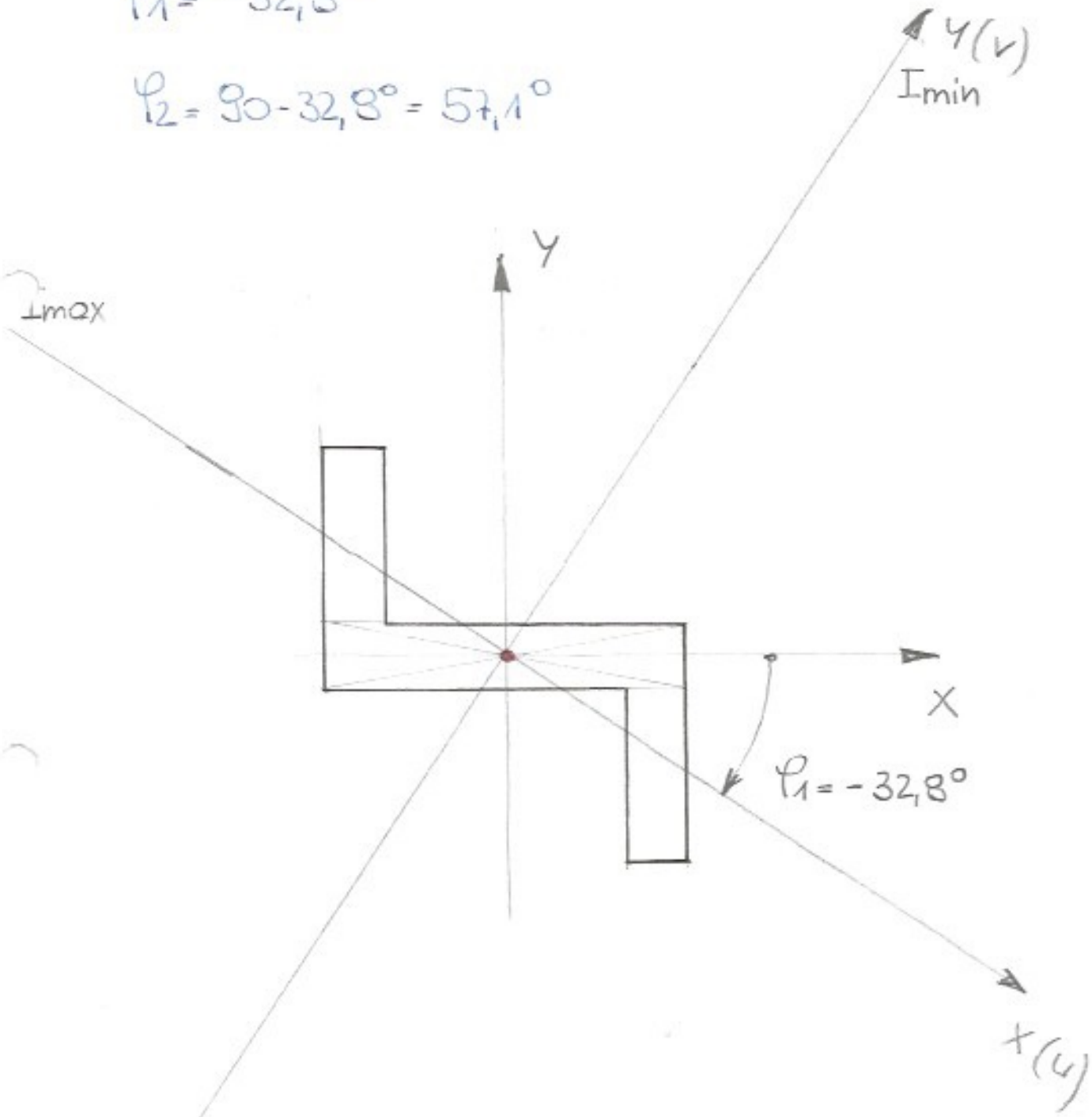
$$I_{\substack{max \\ min}} = 42{,}5 \pm 32{,}89$$

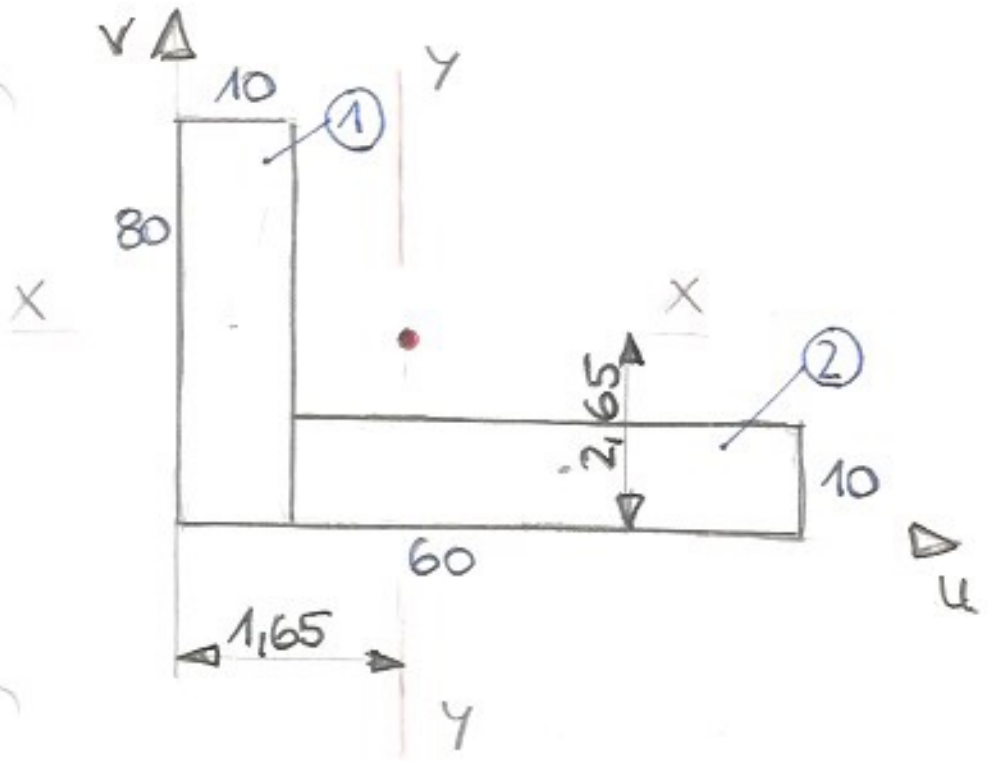
$I_{max} = 75{,}39\ dm^4$
$I_{min} = 9{,}61\ dm^4$

$$\tan 2\varphi = \frac{I_{xy}}{\frac{I_x - I_y}{2}} = \frac{30}{-13,5}$$

$\varphi_1 = -32,8°$

$\varphi_2 = 90 - 32,8° = 57,1°$





1. SP- bestimmen

i	Ai [cm²]	vi [cm]	ui [cm]	Ai vi	Ai ui
1	8	4	0,5	32	4
2	5	0,5	3,5	2,5	17,5
	13			34,5	21,5

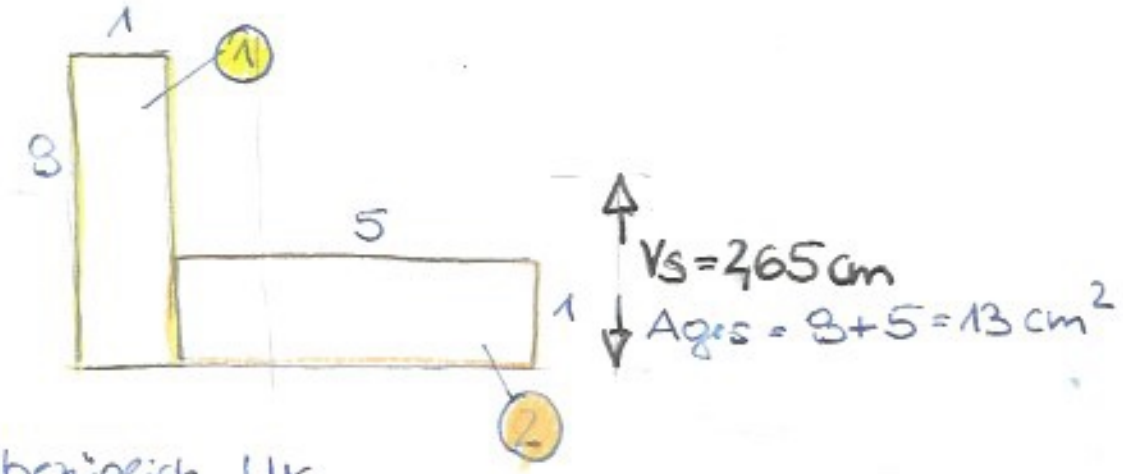
$v_S = \frac{34,5}{13} = \underline{2,65}$ cm

$u_S = \frac{21,5}{13} = \underline{1,65}$ cm

2. I_X

i	I_{xi}	k_i	A_i	$A_i \cdot k_i^2$
1	$\frac{1 \cdot 8^3}{12} = 42{,}66$	$4 - 2{,}65 = 1{,}35$	8	14,58
2	$\frac{5 \cdot 1^3}{12} = 0{,}416$	$2{,}65 - 0{,}5 = 2{,}15$	5	23,11
	Σ 43,07		Σ 37,69	

$I_X = 43{,}07 + 37{,}69 = \underline{80{,}76\ cm^4}$



bezüglich UK

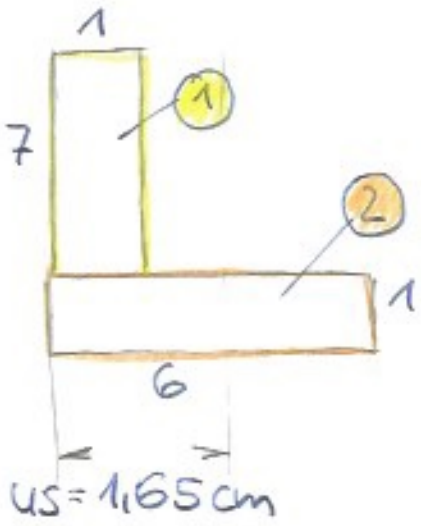
$$I_{uges} = I_{u1} + I_{u2} = \frac{1}{3}\left[1 \cdot 8^3 + 5 \cdot 1^3\right]$$

$$I_{uges} = 172{,}33\overline{3}\ cm^4$$

$$I_X = I_{uges} - k^2 \cdot A = 172{,}33\overline{3} - 2{,}65^2 \cdot 13$$

$$\underline{I_X = 81\ cm^4}$$

3. I_y



λ	I_{yi}	K_i	A_i	$A_i K_i^2$
1	$\frac{7 \cdot 1^3}{12} = 0{,}583$	$1{,}65 - 0{,}5 = 1{,}15$	7	9,257
2	$\frac{1 \cdot 6^3}{12} = 18$	$3 - 1{,}65 = 1{,}35$	6	10,935
	Σ 18,583			20,192

$I_y = 18{,}583 + 20{,}192 = \underline{38{,}775\ cm^4}$

bezüglich UK

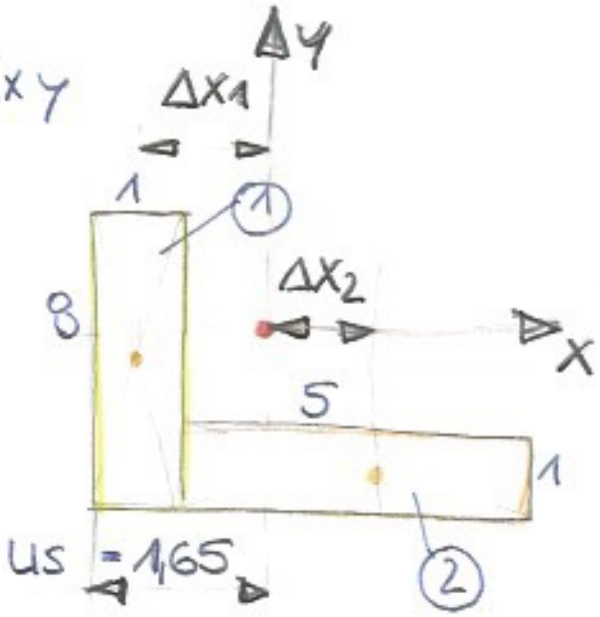
$I_{ges} = I_{u1} + I_{u2} = \frac{1}{3}\left[7 \cdot 1^3 + 1 \cdot 6^3\right]$

$I_{ges} = 74{,}333\ cm^4$

$I_y = I_{ges} - K^2 \cdot A = 74{,}333 - 1{,}65^2 \cdot 13$

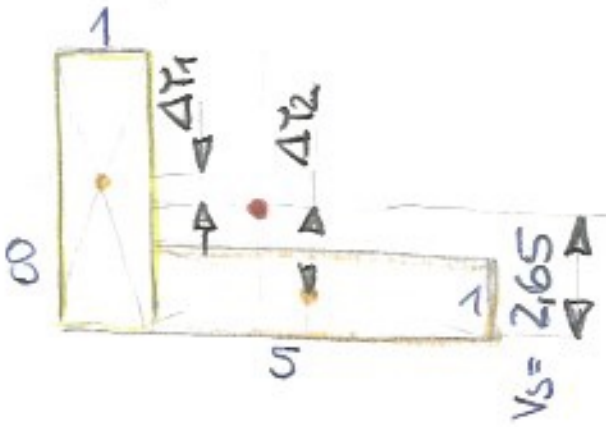
$\underline{I_y = 38{,}94\ cm^4}$

4. I_{xy}



$|\Delta x_1| = us - 0,5$
$= 1,65 - 0,5 = 1,15$

$|\Delta x_2| = 3,5 - 1,65$
$= 1,85$



$|\Delta y_1| = 4 - 2,65$
$= 1,35$

$|\Delta y_2| = 2,65 - 0,5$
$= 2,15$

i	$I_{x'y'}$	A	dx	dy	A dx dy
1	ø	-(8	-1,15	1,35)	12,42
2	ø	-(5	1,85	-2,15)	19,887

Σ 32,3

$I_{xy} = 32,3\ cm^4$

Querschnittseigenschaften der ausgewählten Fläche von Teil1

Fläche = 13.00 Zentimeter^2

Schwerpunkt relativ zum Ursprung des Ausgabekoordinatensystems: (Zentimeter)
X = 1.65
Y = 2.65
Z = 0.00

Trägheitsmomente der Fläche am Schwerpunkt: (Zentimeter ^ 4)

Lxx = 80.78	Lxy = 32.31	Lxz = 0.00
Lyx = 32.31	Lyy = 38.78	Lyz = 0.00
Lzx = 0.00	Lzy = 0.00	Lzz = 119.55

Polares Trägheitsmoment der Fläche am Schwerpunkt = 119.55 Zentimeter ^ 4

Winkel zwischen den Hauptachsen und Teilachsen = 118.49 Grad

Hauptträgheitsmomente der Fläche am Schwerpunkt: (Zentimeter ^ 4)
Ix = 21.24
Iy = 98.31

Trägheitsmomente der Fläche am Ausgabe-Koordinatensystem: (Zentimeter ^ 4)

LXX = 172.33	LXY = 24.75	LXZ = 0.00
LYX = 24.75	LYY = 74.33	LYZ = 0.00
LZX = 0.00	LZY = 0.00	LZZ = 246.67

Hauptachsenflächenmomente

$$I_{1,2} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{1,2} = \frac{80,76 + 38,775}{2} \pm \sqrt{\left(\frac{8976 - 38,775}{2}\right)^2 + 32,3^2}$$

$$I_{12} = 59,76 \pm 38,52$$

$$I_1 = 98,28 \text{ cm}^4$$

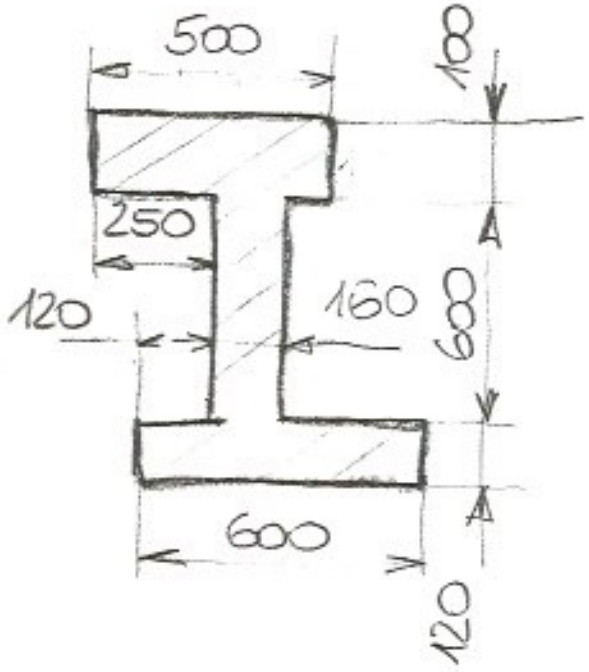
$$I_2 = 21,24 \text{ cm}^4$$

$$\tan 2\varphi_0 = \frac{2 I_{xy}}{I_x - I_y} = \frac{2 \cdot 32,3}{80,76 - 38,775}$$

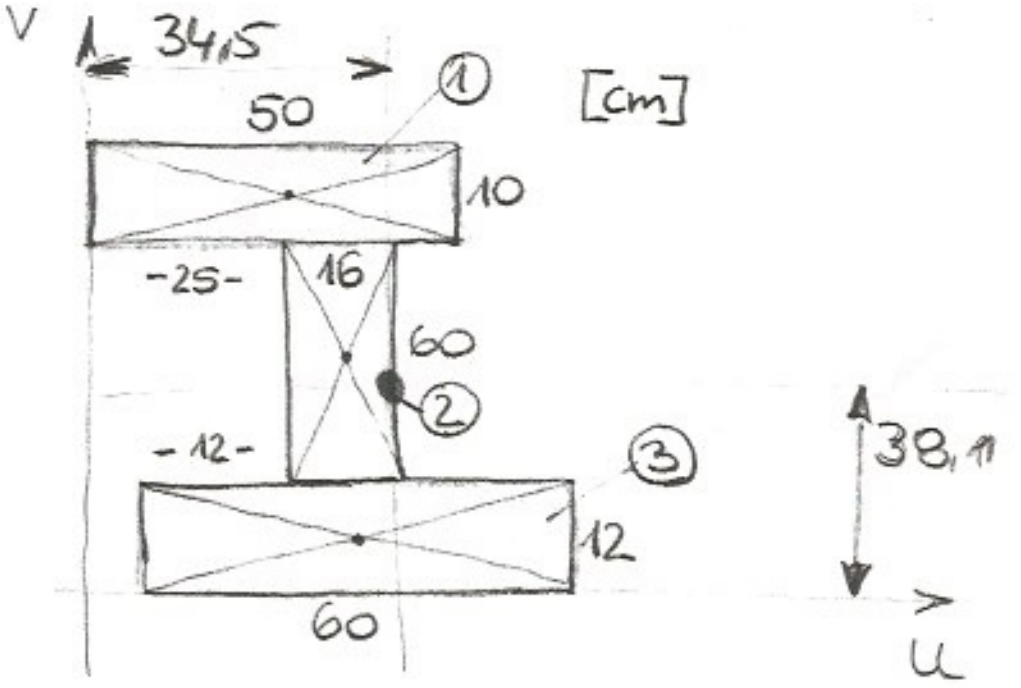
$$\tan 2\varphi_0 = 1,53$$

$$\varphi_0 = 28,48°$$

Bsp.: [Winkler]

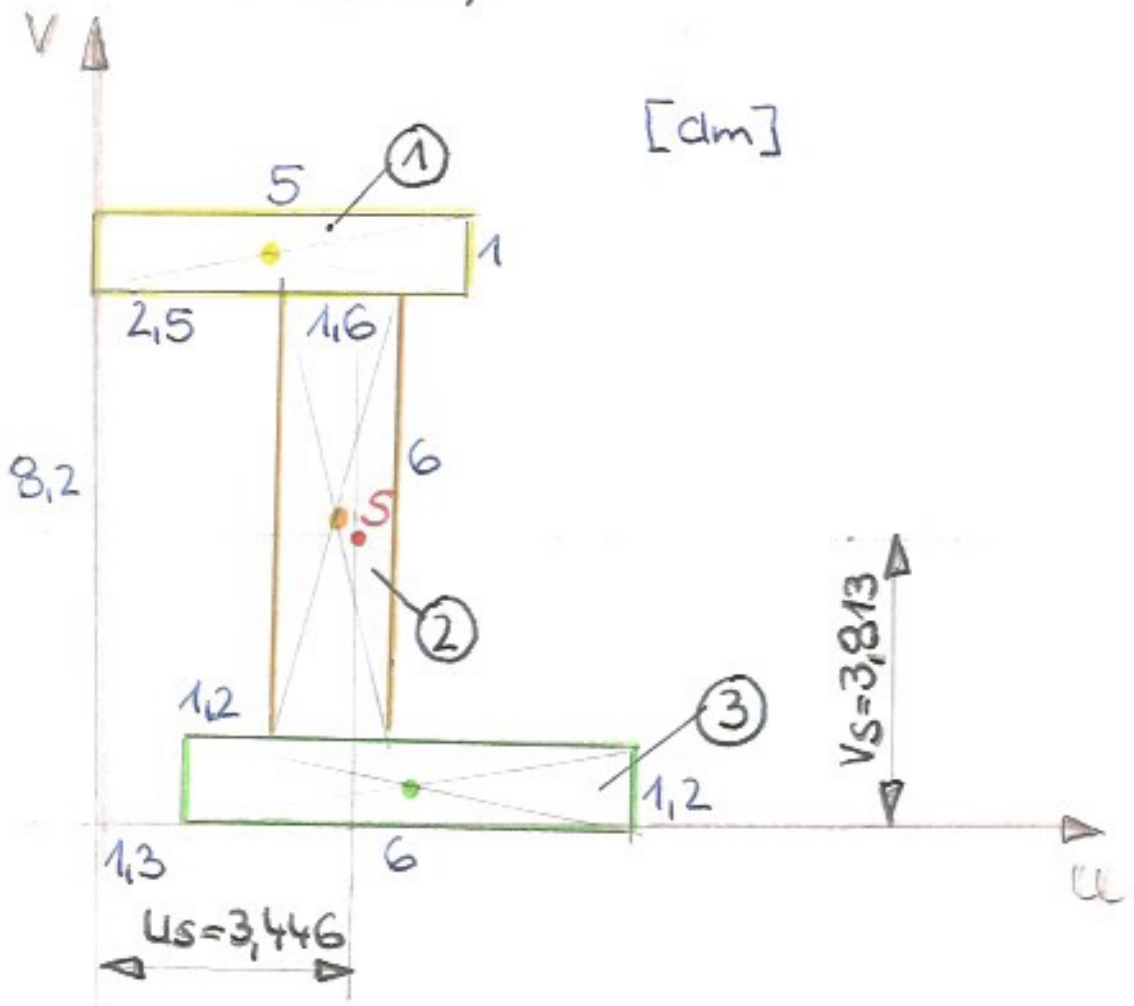


-Hausaufgabe-



Bsp.: Trägheitsmomente

I_x ; I_y ; I_{xy}



1. SP-bestimmen

i	A_i	V_i	U_i	$A_i \cdot V_i$	$A_i \cdot U_i$
1	5	7,7	2,5	38,5	12,5
2	9,6	4,2	3,3	40,32	31,68
3	7,2	0,6	4,3	4,32	30,96
	21,8			83,14	75,14

$$V_S = \frac{83,14}{21,8} = \underline{3,813 \text{ dm}}$$

$$U_S = \frac{75,14}{21,8} = \underline{3,446 \text{ dm}}$$

2. I_x

i	I_{xi}	A_i	K_i	$A_i K_i^2$
1	$\frac{5 \cdot 1^3}{12} = 0{,}416$	5	$7{,}7 - 3{,}813 = 3{,}88$	75,272
2	$\frac{1{,}6 \cdot 6^3}{12} = 28{,}8$	9,6	$4{,}2 - 3{,}813 = 0{,}387$	1,437
3	$\frac{6 \cdot 1{,}2^3}{12} = 0{,}864$	7,2	$3{,}813 - 0{,}6 = 3{,}213$	74,328

$\sum$ 30,08 + 151,037

$I_x = 181{,}1 \text{ dm}^4$

3. I_y

λ	I_{yi}	A_i	K_i	$A_i \cdot K_i^2$
1	$\frac{1 \cdot 5^3}{12} = 10{,}416$	5	$3{,}446 - 2{,}5 = 0{,}946$	4,474
2	$\frac{6 \cdot 1{,}6^3}{12} = 2{,}04$	9,6	$3{,}446 - 3{,}3 = 0{,}146$	0,204
3	$\frac{1{,}2 \cdot 6^3}{12} = 21{,}6$	7,2	$4{,}3 - 3{,}446 = 0{,}854$	5,25
	Σ 34,056			9,928

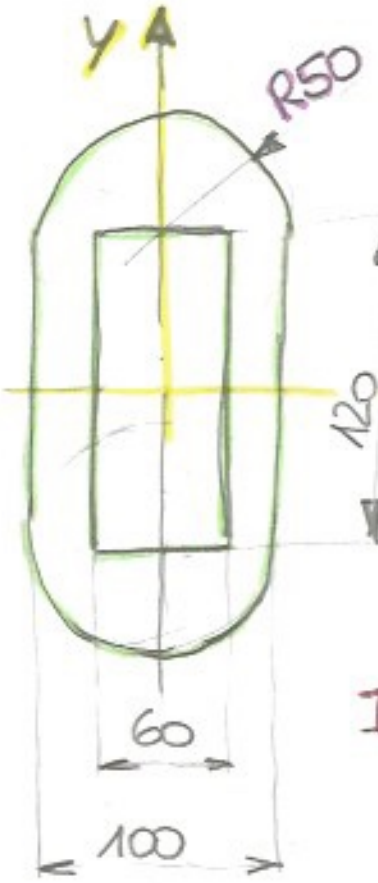
$\underline{I_y = 43{,}984}\ dm^4$

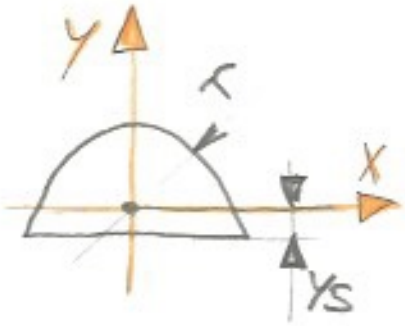
4. I_{xy}

i	$I_{x'y'}$	dx	dy	Ai	Ai dx dy
1	ø	-0,946	+3,88	5	-18,352
2	ø	-0,146	+0,387	9,6	-0,5424
3	ø	+0,854	-3,21	7,2	-19,737
					Σ -38,63

$$I_{xy} = 38{,}63 \text{ dm}^4$$

Flächenhägheitsmoment I_x, I_y



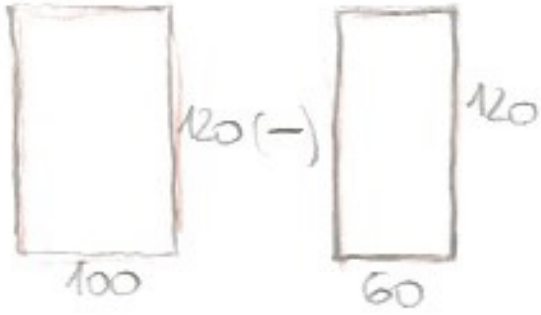


$$y_s = \frac{4 \cdot r}{3\pi}$$

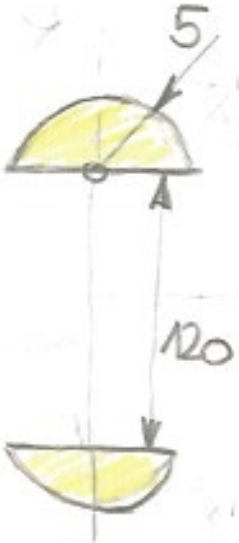
$$A = \frac{\pi}{8} d^2$$

$$I_{xges} = I_{x1} - I_{x2} + 2I_{xHcl}$$

I_{x-x}

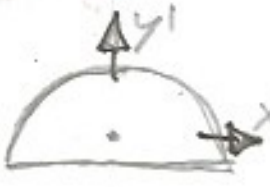


+



Reduzierung der Schwerpunkte auf die Halbkreise

$$I_x = I_y = \frac{\pi}{64} d^4$$

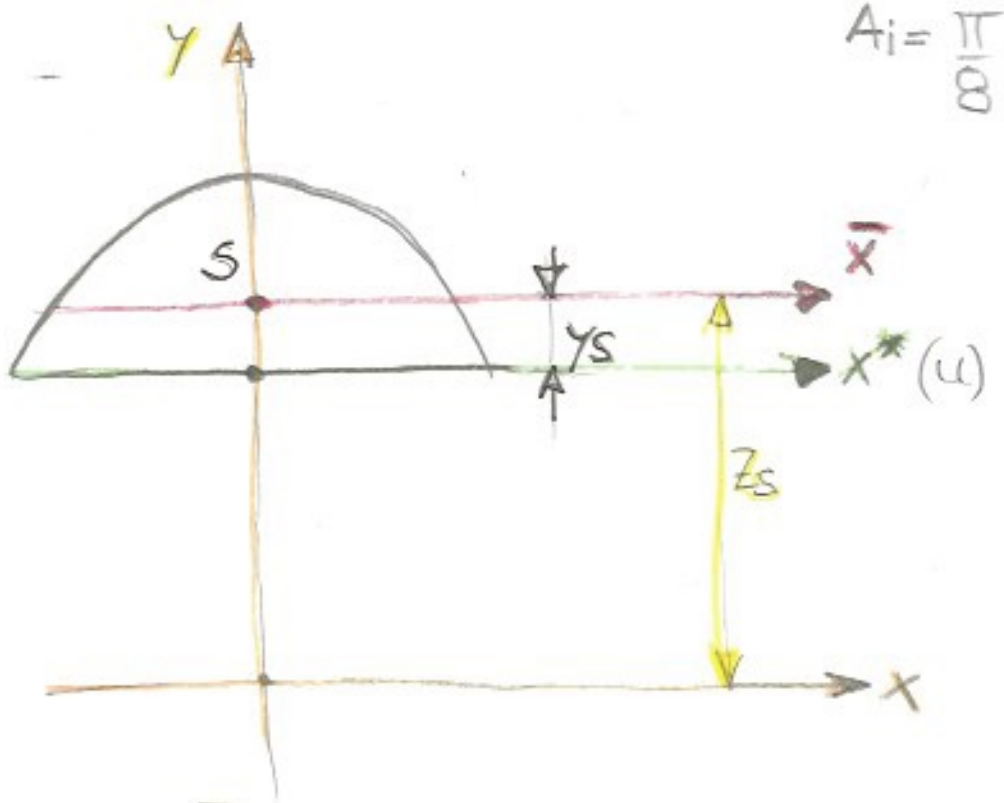


i	Ix	ℓ_i	$\ell_i^2 \cdot A_i$
1	245,437	2,122	176,829

$\ell_i \rightarrow$

$y_S = \frac{4 \cdot 5}{3\pi} = 2{,}122\,cm$

$A_i = \frac{\pi}{8} \cdot 10^2 = 39{,}27\,cm^2$



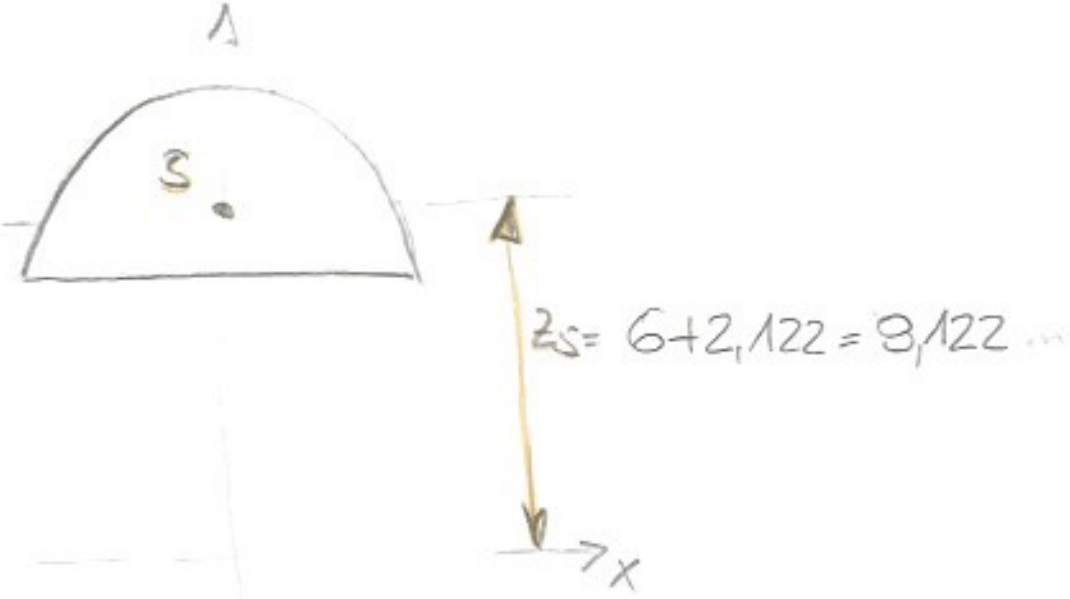
$$I_{x^*} = \frac{\pi}{128} d^4 = 245{,}4\,cm^4$$

$$I_{x^*} = \bar{I}_x + y_S^2 \cdot A = I_u$$

$$\bar{I}_x = I_x^* - y_S^2 \cdot A = 245{,}4\,cm^4 - 176{,}828$$

$$\bar{I}_x = 68{,}572\,cm^4$$

Reduzierung auf die Hauptträgheitsachse



$$I_{x\,red} = \bar{I}_x + z_S^2 \cdot A = 2659{,}092\ cm^4$$

1	$\bar{I}_x$	$z_S^2 \cdot A$
1	68,572	2590,52

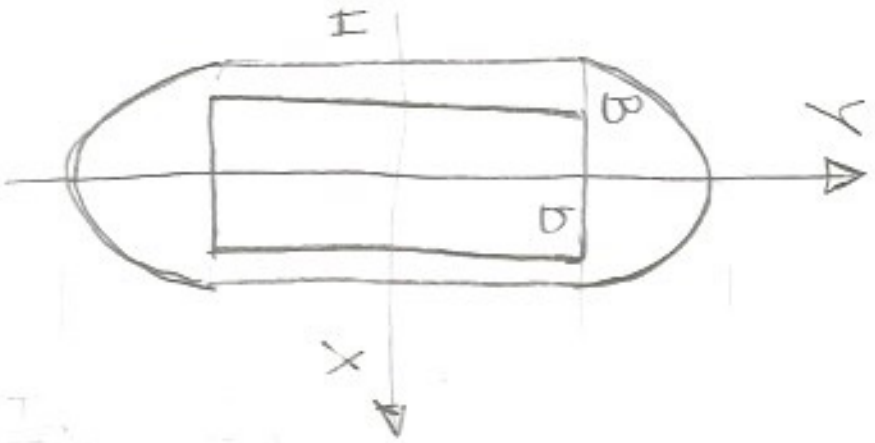
$$I_{ges} = I_{x1} - I_{x2} + 2\,I_{x\,red}$$

$$= \frac{B\,H^3}{12} - \frac{b\,H^3}{12} + 2\,I_{x\,red}$$

$$= \frac{H^3}{12}(B-b) + 2 \cdot 2659\ cm^4$$

$$= \frac{12^3}{12}(4) + 2 \cdot 2659 = 5894\ cm^4$$

I_{y-y}



$$I_{ges} = I_{\bigcirc} + I_{\square} - I_{\square}$$

$$I_{ges} = \frac{\pi}{64} d^4 + \frac{H \cdot B^3}{12} - \frac{H \cdot b^3}{12}$$

$$= \frac{\pi}{64} 10^4 + \frac{H}{12} (B^3 - b^3)$$

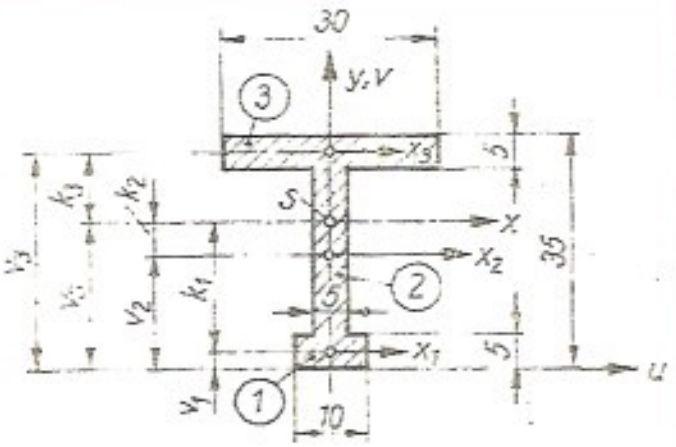
$$= \frac{\pi}{64} 10^4 + \frac{12}{12} (10^3 - 6^3)$$

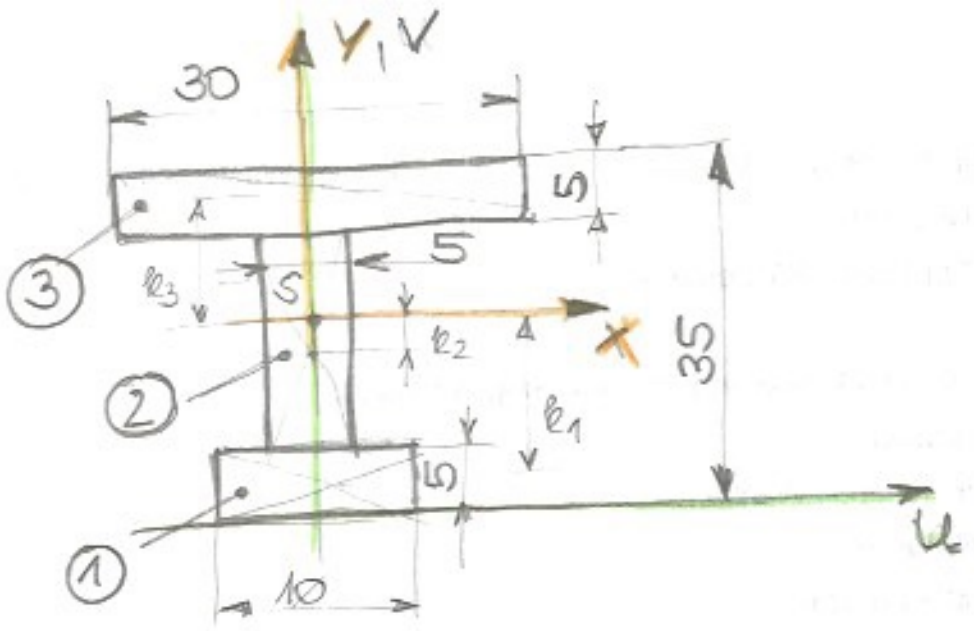
$$I_{ges} = 1274{,}87 \text{ cm}^4$$

2. Aufgabe (.. Punkte) :

Der vorliegende Querschnitt (**Bild 2**)
ist einfachsymmetrisch
(y - Achse als Symmetrieachse).
Berechnen Sie :

1. Den Flächenschwerpunkt v_s (S).
2. Das Flächenträgheitsmoment zweiter Ordnung bezüglich der X-Achse.
3. Das Flächenträgheitsmoment zweiter Ordnung bezüglich der Y-Achse.





1. Schwerpunkt bestimmen:

$$v_S = \frac{A_1 \cdot v_1 + A_2 \cdot v_2 + A_3 \cdot v_3}{A_1 + A_2 + A_3}$$ (1P)

$A_{n1} = 9{,}5 \cdot 1 = 9{,}5\,cm^2$
$A_2 = 2{,}5 \cdot 0{,}5 = 1{,}25\,cm^2$
$A_3 = 3 \cdot 0{,}5 = 1{,}5\,cm^2$

$$v_S = \frac{9{,}5 \cdot 0{,}25 + 1{,}25 \cdot (1{,}25 + 0{,}5) + 1{,}5 \cdot (3 + 0{,}25)}{9{,}5 + 1{,}25 + 1{,}5}$$

$v_S = 2{,}212\,cm$ (1P)

2. Hauptträgheitsmomente bestimmen

$$I_{x1} = \frac{b \cdot h^3}{12} = \frac{1 \cdot 9{,}5^3}{12} = 0{,}01\,cm^4$$

$$I_{x2} = \frac{b \cdot h^3}{12} = \frac{9{,}5 \cdot 2{,}5^3}{12} = 0{,}651\,cm^4$$ (1,5)

$$I_{x3} = \frac{b \cdot h^3}{12} = \frac{3 \cdot 9{,}5^3}{12} = 0{,}031\,cm^4$$