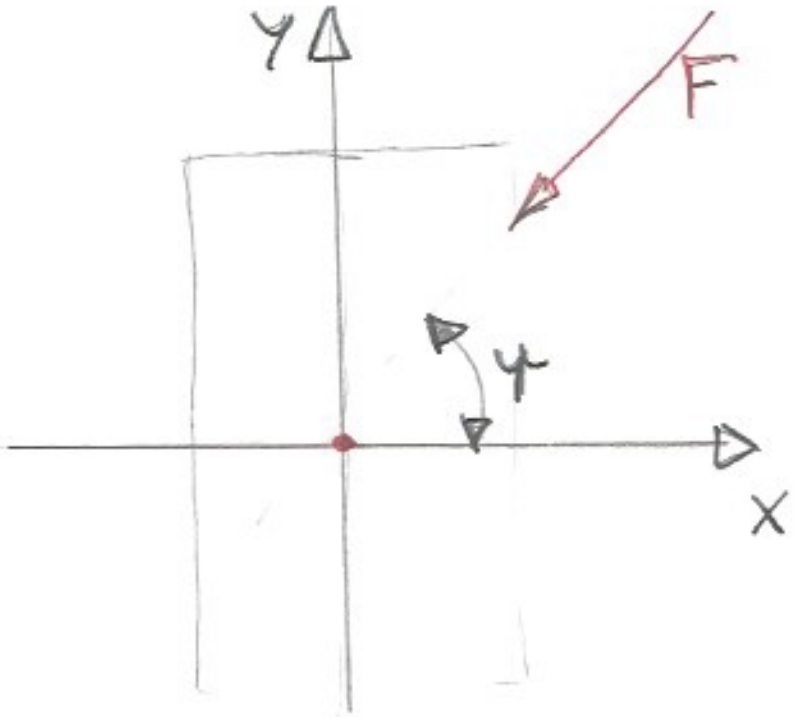
8.05.2011



$$F_X = F \cdot \cos\varphi$$

$$F_y = F \cdot \sin\varphi$$

$$M_X = + F_y \cdot e = F \cdot \sin\varphi \cdot e$$

$$M_y = - F_X \cdot e = - F \cdot \cos\varphi \cdot e$$

$$M_{res} = \sqrt{(-F_X \cdot e)^2 + (F_y \cdot e)^2}$$

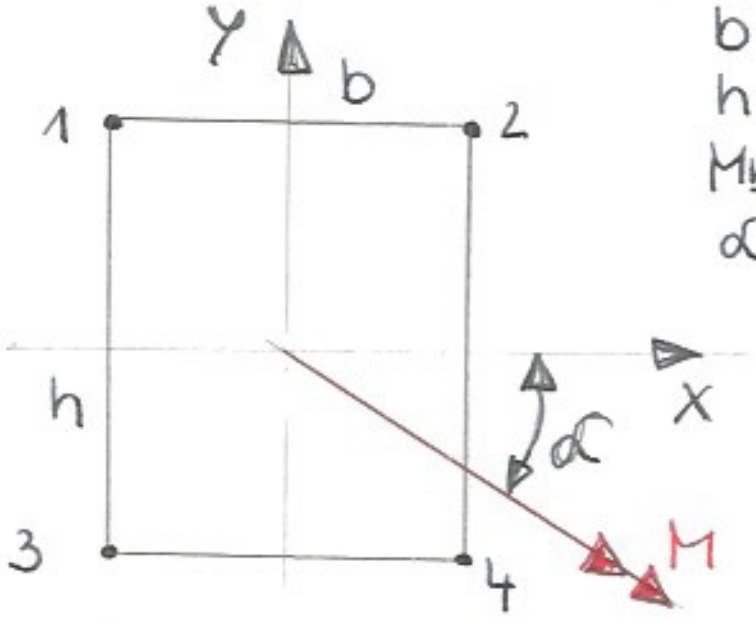
$$M_{res} = \sqrt{F^2 \cdot e^2 \cdot \left[(-\cos^2\varphi) + (\sin^2\varphi)\right]}$$

$$\sqrt{F^2 \cdot \sin^2\varphi \cdot e^2 + -F^2 \cdot \cos^2\varphi \cdot e^2}$$

$$\sqrt{F^2 \cdot e^2 \left(\sin^2\varphi + \cos^2\varphi\right)} = F \cdot e \qquad \text{W.z.b.w.}$$

gege.:

$b = 4\,cm$
$h = 6\,cm$
$M_b = 1\,KNm$
$\alpha = 30°$



ges.:

- $\sigma_{b1} \dots \sigma_{b4}$
- $y = f(x)$ SpnLV

Lösung:

$$I_x = \frac{b \cdot h^3}{12} = \frac{4 \cdot 6^3}{12} = 72\,cm^4$$

$$I_y = \frac{h \cdot b^3}{12} = \frac{6 \cdot 4^3}{12} = 32\,cm^4$$

$$\sigma_b = \frac{M_x}{I_x} y - \frac{M_y}{I_y} \cdot x$$

$$\sigma_b = \frac{M \cdot \cos\alpha}{I_x} y + \frac{M \cdot \sin\alpha}{I_y} x$$

$M_b = 1\ KNm = 100\ KNcm$

$$\sigma_{bi} = \frac{100 \cdot \cos 30^\circ}{72} y + \frac{100 \sin 30^\circ}{32} x$$

$$\sigma_{bi} = 1{,}2\, y + 1{,}56\, x \quad \left[\frac{KN}{cm^2}\right]$$

$\sigma_{b1} = 1{,}2\, y_1 + 1{,}56\, x_1$ $\quad P_1(-2;\ 3)$

$\sigma_{b2} = 1{,}2\, y_2 + 1{,}56\, x_2$ $\quad P_2(2;\ 3)$

$\sigma_{b3} = 1{,}2\, y_3 + 1{,}56\, x_3$ $\quad P_3(-2;\ -3)$

$\sigma_{b4} = 1{,}2\, y_4 + 1{,}56\, x_4$ $\quad P_4(2;\ -3)$

$\sigma_{b1} = 1{,}2 \cdot 3 + 1{,}56 \cdot (-2) = 0{,}48\ \frac{KN}{cm^2} = 4{,}8\ \frac{N}{mm^2}$

$\sigma_{b2} = 1{,}2 \cdot 3 + 1{,}56 \cdot 2 = 6{,}72\ \frac{KN}{cm^2} = 67{,}2\ \frac{N}{mm^2}$

$\sigma_{b3} = 1{,}2 \cdot (-3) + 1{,}56 \cdot (-2) = -6{,}72\ \frac{KN}{cm^2} = -67{,}2\ \frac{N}{mm^2}$

$\sigma_{b4} = 1{,}2(-3) + 1{,}56 \cdot 2 = -0{,}48\ \frac{KN}{cm^2} = -4{,}8\ \frac{N}{mm^2}$

Spannungsnullinie

$y = -\frac{1,56}{1,2} x \quad -1,2x \qquad \alpha = -52,43°$

$$\sigma = \varnothing = \frac{1039}{567,76} \cdot y + \frac{600}{1373,16} x$$

$$y = \varnothing = 1,83 y + 0,437 x$$

$$y = -0,238 x$$

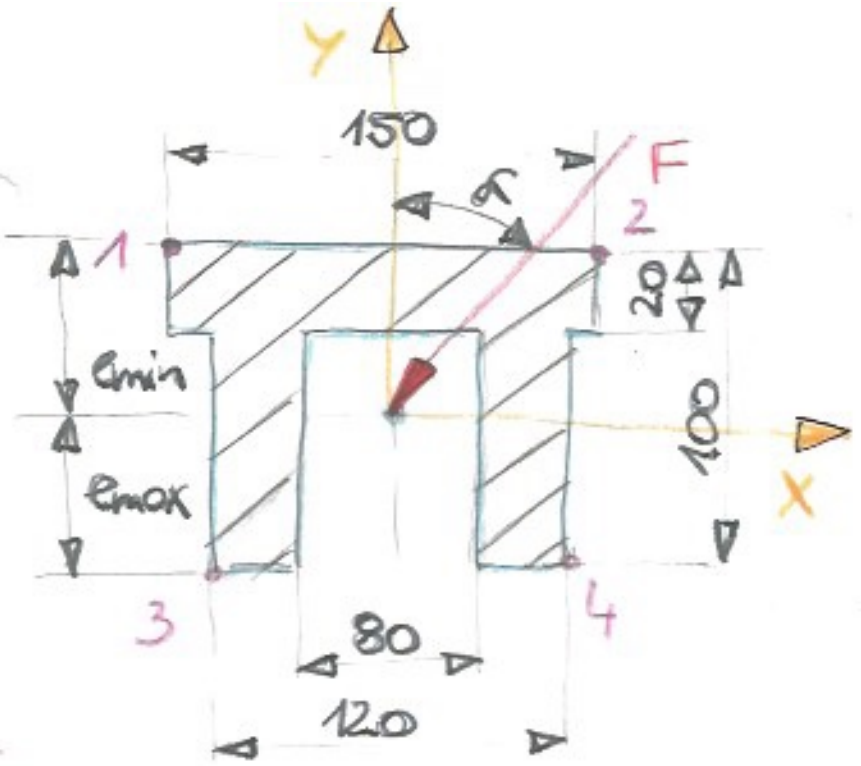
bei $F_{zul} = 60\,KN$

$M_x = 10,38\,KNm$

$M_y = 6\,KNm$

$M_x = 1039\,KNcm$

$M_y = 600\,KNcm$



gegeb.:

$l = 200\,mm$

E295 (St50-2)

$\sigma_{zul} = 145\ \frac{N}{mm^2}$

$\alpha = 30°$

ges.: F_{max}

$I_x = 567{,}76\,cm^4$

$I_y = 1373{,}16\,cm^4$

$e_{min} = 3{,}58\,cm$

$e_{max} = 6{,}42\,cm$

allg.

$$\sigma_{b\,max} = \frac{M_{b\,max}}{I_x}\, e_{max}$$

$$M_{b\,max} = \sigma_{b\,zul}\,\frac{I_x}{e_{max}}$$

Lösung:

$$\sigma_b = \frac{M_x}{I_x}\,y - \frac{M_y}{I_y}\cdot x$$

$$F_y = \cos\alpha \cdot F$$

$$F_x = \sin\alpha \cdot F$$

$$M_x = F_y \cdot l = F\cdot\cos\alpha\cdot l$$

$$M_y = -F_x \cdot l = -F\cdot\sin\alpha\cdot l$$

1.... Zug / Druck

2... Zug / Zug * kritische Punkte

3... Druck / Druck *

4... Druck / Zug

Koordinaten

$P_1 (-7{,}5; +3{,}58)$ [cm]

$P_2 (+7{,}5; +3{,}58)$

$P_3 (-6; -6{,}42)$

$P_4 (6; -6{,}42)$

Untersuchung der Punkte 2 u. 3

$$\sigma_b = \frac{F \cdot \cos\alpha \cdot e}{I_x} \cdot y - \frac{-F \cdot \sin\alpha \cdot e}{I_y} x$$

$$\sigma_b = F \frac{\cos\alpha \cdot e \cdot y}{I_x} + \frac{\sin\alpha \cdot e \cdot x}{I_y}$$

$$F_{max} = \frac{\sigma_b}{\frac{\cos\alpha \cdot e}{I_x} y + \frac{\sin\alpha \cdot e}{I_y} \cdot x}$$

$$F_{max} = \frac{145\,00}{\frac{\cos 30° \cdot 20}{567{,}76} y + \frac{\sin 30° \cdot 20}{1373{,}16} x}$$

$$F_{max} = \frac{14.500}{0{,}030 \cdot y + 0{,}00728 x}$$

$x = 7{,}5$; $y = 3{,}58$; $F_{max} = 89{,}5$ kN

$x = -6$; $y = -6{,}42$; $F_{max} = 61{,}3$ kN

Festlegung $F_{zul} = 60$ kN

Berechnung der Randspannungen

$$\sigma_i = \frac{M_x}{I_x} y_i - \frac{M_y}{I_y} x_i$$

$$M_x = + F \cdot \cos 30° \cdot \ell$$
$$= 60 \cdot 10^3 N \cdot \cos 30° \cdot 20\,cm$$
$$= 60 \cdot \cos 30° \cdot 20$$
$$= 1039{,}23\ kNcm$$

$$M_y = - F \cdot \sin 30° \cdot \ell$$
$$= -60 \cdot \sin 30° \cdot 20$$
$$= -600\ kN\,cm$$

$$\sigma_i = \frac{1039{,}23\ kNcm}{567{,}76\ cm^4} y_i - \frac{-600\ kNcm}{1373{,}16\ cm^4} \cdot x_i$$

$$\sigma_i = 1{,}83\, y_i + 0{,}4369\, x_i \quad \left[\frac{kN}{cm^2}\right] \cdot \frac{10^3}{10^2} \left[\frac{N}{mm^2}\right]$$

$\sigma_1 (-7{,}5; +3{,}58) = 3{,}27 \frac{kN}{cm^2} = \underline{32{,}7\ N/mm^2}$

$\sigma_2 (7{,}5; +3{,}58) = 9{,}8 \frac{kN}{cm^2} = \underline{98\ N/mm^2}$

$\sigma_3 (-6; -6{,}42) = -14{,}37 \frac{N}{cm^2} = \underline{-143{,}7\ N/mm^2}$

$\sigma_4 (6; -6{,}42) = -9{,}127 \frac{N}{cm^2} = \underline{-91{,}27\ N/mm^2}$

Spannungsnullinie

$$y = -\frac{0{,}4369}{1{,}83} x$$

$$y = -0{,}238\, x \qquad \alpha = -13{,}38°$$

Querschnittskennwerte

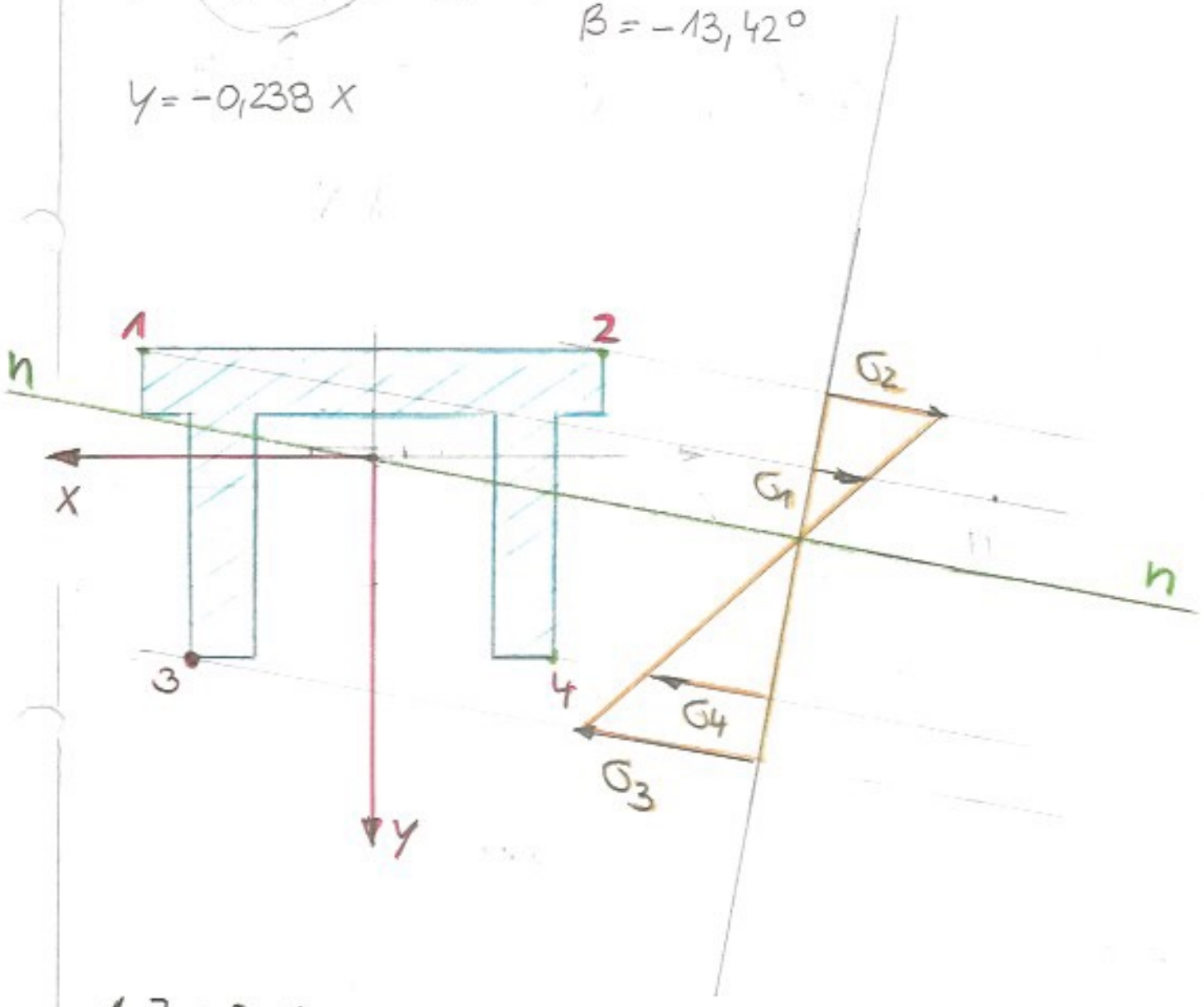
$e_{y\,min} = 35{,}8\ mm$ $\qquad I_x = 567{,}7634\ cm^4$

$e_{y\,max} = 64{,}2\ mm$ $\qquad I_y = 1373{,}16\overline{6}\ cm^4$

$$y = \frac{I_x}{I_y} \cdot \tan\alpha \cdot x \;;\quad \tan\beta = -0{,}2387$$

$$\beta = -13{,}42°$$

$$y = -0{,}238\,x$$

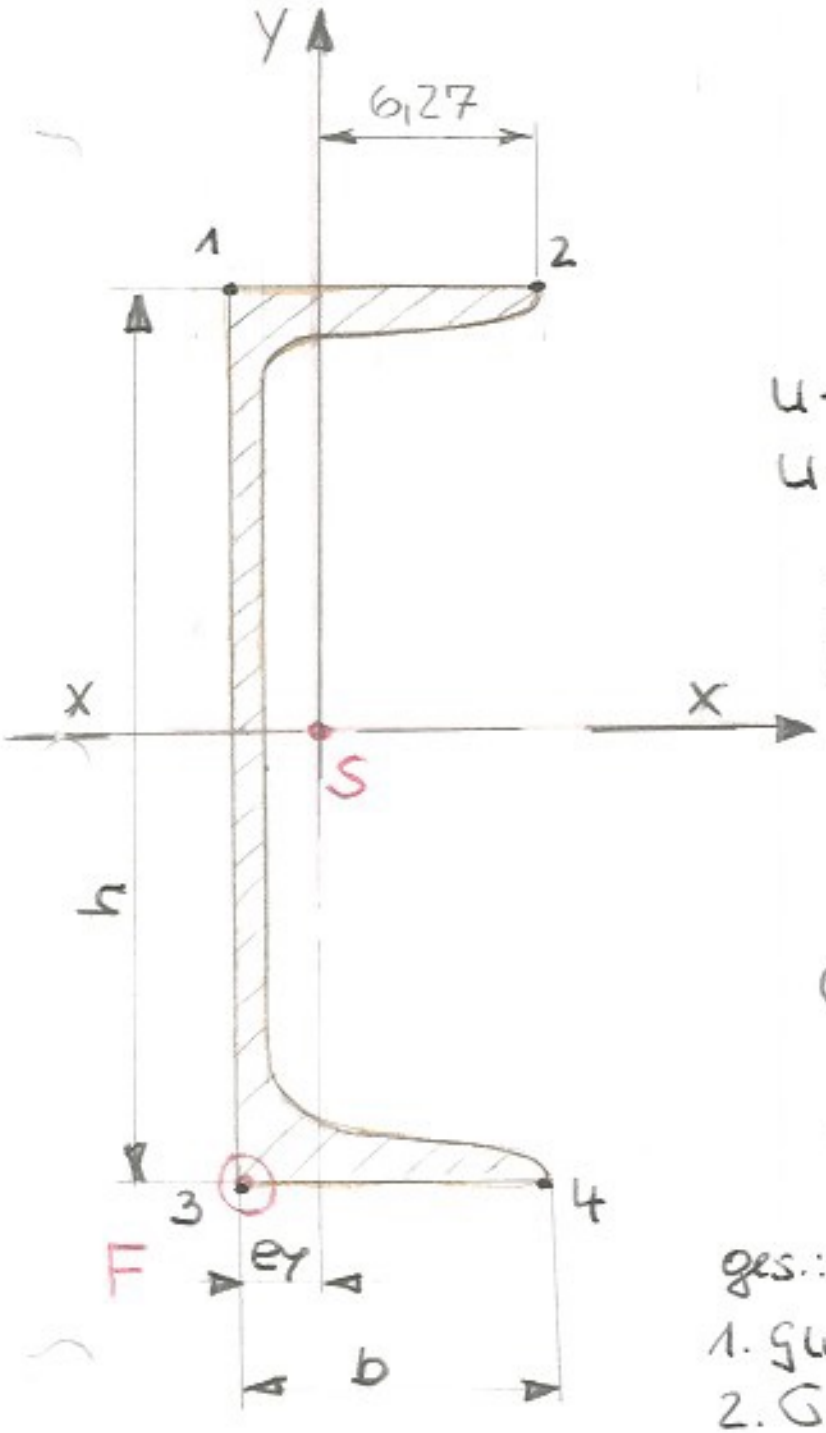


1 – Zug, Druck

2 – Zug, Zug

3 – Druck, Druck → $|\sigma_3| = \sigma_{max}$

4 – Druck, Zug



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U-240

$I_X = 3600\,cm^4$

$I_y = 248\,cm^4$

$e_y = 2{,}23\,cm$

$b = 8{,}5\,cm$

$A = 42{,}3\,cm^2$

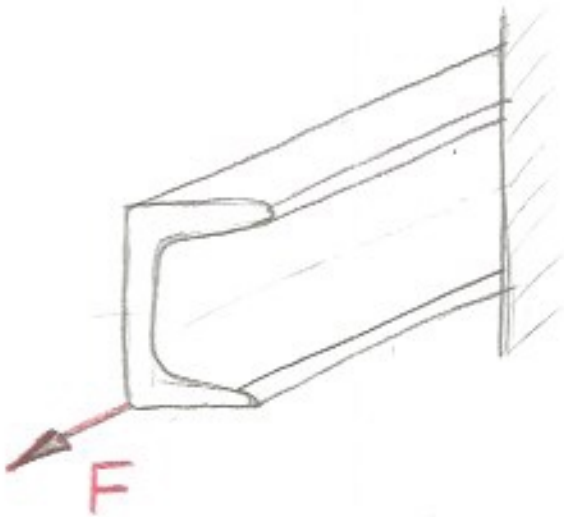
$h = 24\,cm$

$\sigma_{zul} = 160\ \frac{N}{mm^2}$

$F = 180\,kN$

ges.:

1. glng. Spannungsnulllinie
2. σ_{ges1} ... σ_{ges4}
3. Graphik-Spannungsverlauf



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Gesucht:

1. Gleichung der Spannungsnulllinie
2. σ_{rcs1} bis σ_{rcs4}
3. Grafische Darstellung der Spannungen

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zu 1. $\sigma_{rcs} = \emptyset = \sigma_z + \sigma_b$

$$\sigma_{rcs} = \frac{F}{A} + \frac{M_x}{I_x} y - \frac{M_y}{I_y} x = \emptyset$$

$$M_x = -F \cdot \frac{h}{2} = -180\,KN \cdot 12\,cm = \underline{-2160\,KNcm}$$

$$M_y = +F \cdot e_y = 180\,KN \cdot 2{,}23\,cm = \underline{401{,}4\,KNcm}$$

$$\emptyset = \frac{180\,KN}{42{,}3\,cm^2} + \frac{-2160\,KNcm}{3600\,cm^4} y - \frac{401{,}4\,KNcm}{248\,cm^4} x$$

$$\emptyset = 4{,}25 - 0{,}6y - 1{,}618x$$

$$y = -2{,}69x + 7{,}08 \quad [cm]$$

Einheiten: $\frac{KN}{cm^2} \cdot \frac{cm^4}{KNcm} = [cm]$

Zu 2.

$$\sigma_{HCS} = \frac{F}{A} + \frac{M_X}{I_X} y - \frac{M_Y}{I_Y} x$$

$P_1 (-2{,}23;\ 12)$
$P_2 (6{,}27;\ 12)$
$P_3 (-2{,}23;\ -12)$
$P_4 (6{,}27;\ -12)$

$$\sigma_{HCS} = \frac{180\,KN}{42{,}3\,cm^2} - \frac{2160\,KNcm}{3600\,cm^4} y - \frac{401{,}4\,KNcm}{248\,cm^4} x$$

$$\sigma_{HCS} = 4{,}255 - 0{,}6y - 1{,}61x$$

$\sigma_{HCS\,1} = 0{,}645\ KN/cm^2 = {}^{+}6{,}45\ N/mm^2$
$\sigma_{HCS\,2} = -13\ KN/cm^2 = -130{,}4\ N/mm^2$
$\sigma_{HCS\,3} = 15{,}04\ KN/cm^2 = 150{,}4\ N/mm^2$
$\sigma_{HCS\,4} = 1{,}36\ KN/cm = 13{,}6\ N/mm^2$

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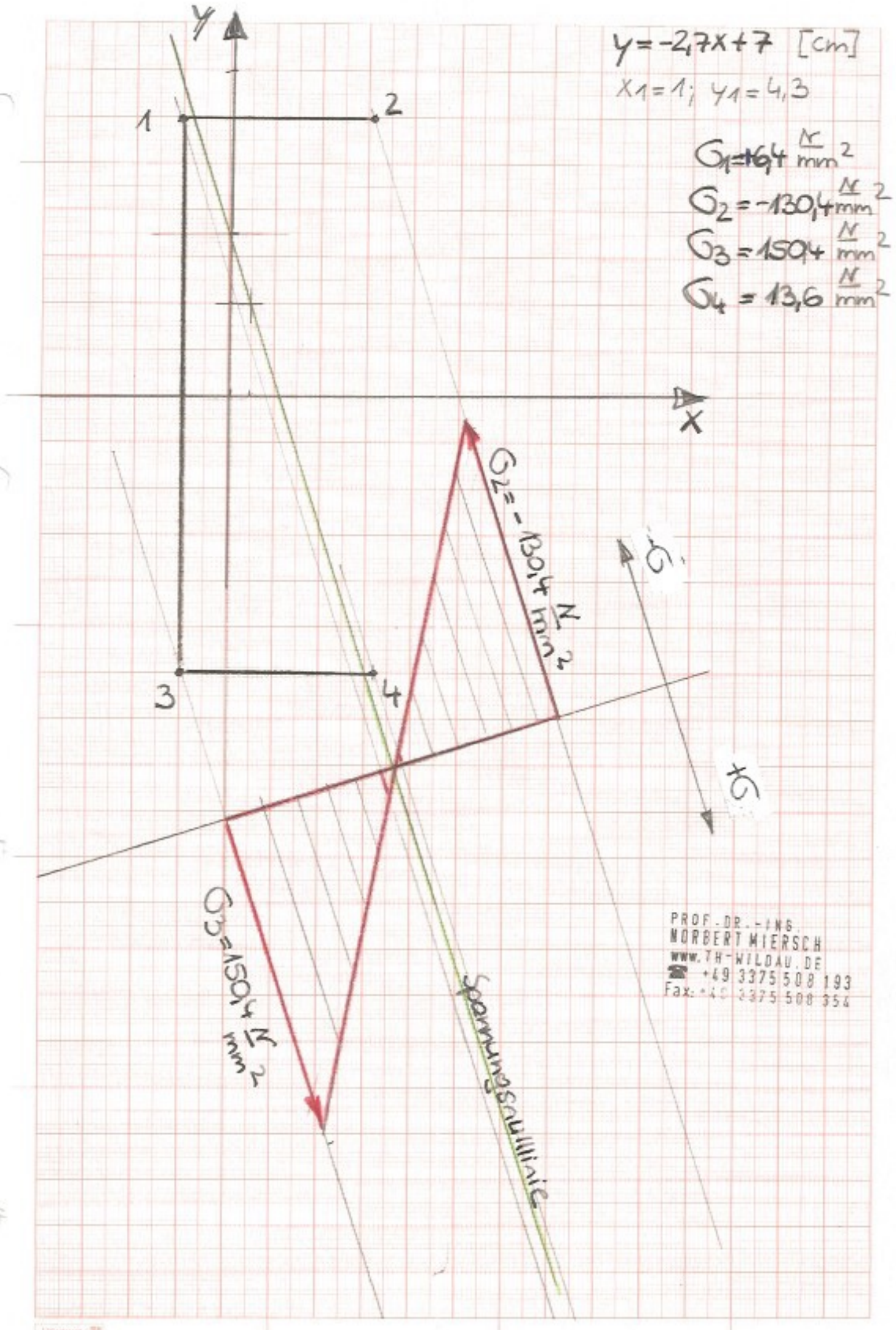
y
x
1
2
3
4
$y = -2{,}7x + 7$ [cm]
$x_1 = 1$; $y_1 = 4{,}3$
$\sigma_1 = 164 \frac{N}{mm^2}$
$\sigma_2 = -130{,}4 \frac{N}{mm^2}$
$\sigma_3 = 1504 \frac{N}{mm^2}$
$\sigma_4 = 13{,}6 \frac{N}{mm^2}$
$\sigma_2 = -130{,}4 \frac{N}{mm^2}$
$\sigma_3 = 1504 \frac{N}{mm^2}$
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