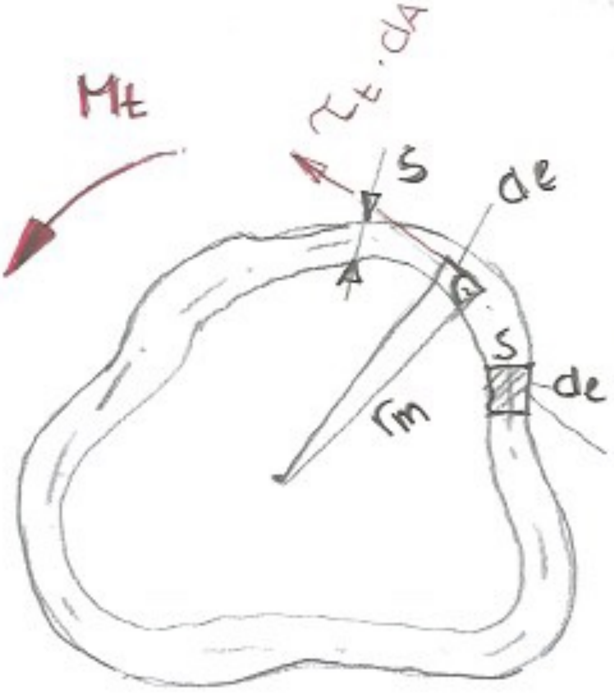
18.05.2015
24.05.2020



$dA_m = \frac{1}{2} r_m \, de$

$dA = s \cdot de$

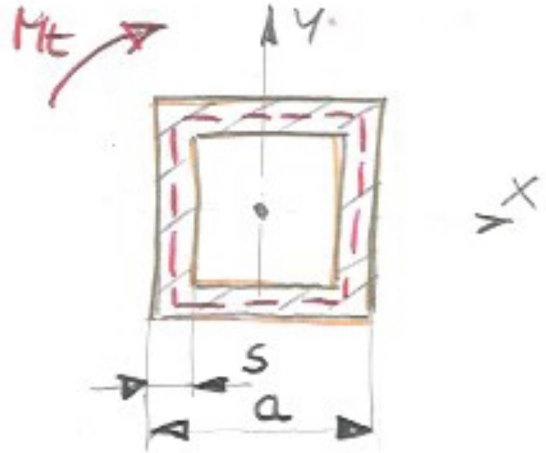
$r_m = \frac{dA_m 2}{de}$

$$dM_t = F \cdot r_m$$
$$= \tau_t \cdot r_m \cdot dA$$
$$= \tau_t \cdot r_m \cdot s \cdot de$$
$$= \tau_t \cdot \frac{dA_m}{\cancel{de}} \, s \cdot \cancel{de} \cdot 2$$
$$M_t = \tau_t \cdot 2 \cdot s \int_{(A)} dA_m$$

Schubfluss $T = \tau_t \cdot s_{min} = konst.$

$$\tau_t = \frac{M_t}{2 \cdot s \cdot A_m}$$ 1. Bredtsche Formel

Lehrbeispiel 77 (Winkler)



gege.: Torsionsstab mit symmetr. Kastenprofil

s = konst.

$a = 100\,mm$

$M_t = 1000\,Nm$

$\tau_{t\,zul} = 20\,N/mm^2$

ges.: s

Lösung:

$$\tau_t = \frac{M_t}{W_t} = \frac{M_t}{2 A_m \cdot s} \qquad W_t \approx 2 \cdot A_m \cdot s$$

zunächst Entwurf mit $A_a = a^2$

$$s = \frac{M_t}{2 \cdot \tau_t \cdot A_a} = \frac{1000 \cdot 10^3\,Nmm^3}{2 \cdot 20\,N \cdot 100^2\,mm^2} = 2{,}5\,mm$$

Nachrechnung mit A_m

$$A_m = (a-s)^2 = 97{,}5^2\,mm^2$$

$$\tau_t = \frac{M_t}{2 \cdot A_m \cdot s} = \frac{1000 \cdot 10^3}{2 \cdot 97{,}5^2 \cdot 2{,}5} = 21\,\frac{N}{mm^2} \approx 20\,\frac{N}{mm^2}$$

28.05.2012
Pfingsten
24. Mai 2020

$$\tau_t = \frac{M_t}{W_t} = \frac{M_t}{2A_m \cdot s}$$

$$A_m = (a-s)^2$$

$$\tau_t = \frac{M_t}{2(a-s)^2 \cdot s}$$

$$2(a-s)^2 \cdot s = \frac{M_t}{\tau_{t\,zul}}$$

$$2(a-s)^2 \cdot s = \frac{1000\,000}{20}$$

$$2(a-s)^2 \cdot s = 50\,000$$

$$(a-s)^2 \cdot s = 25\,000$$

$$(a^2-2as+s^2) \cdot s = 25\,000$$

$$a^2 \cdot s - 2as^2 + s^3 = 25\,000$$

$$s^3 - 2as^2 + a^2 \cdot s - 25\,000 = 0$$

$$s^3 - 200s^2 + 10\,000s - 25\,000 = 0$$

$$s_3 = 82{,}6 \text{ mm}$$

Hornersches Schema

	1	-200	10 000	-25 000
82,6		82,6	-9697,24	25 000
82,6	1	-117,4	302,76	0

$$s^2 - 117{,}4s + 302{,}76 = 0$$

$$s_{21} = 58{,}7 \pm \sqrt{3445{,}69 - 302{,}76}$$

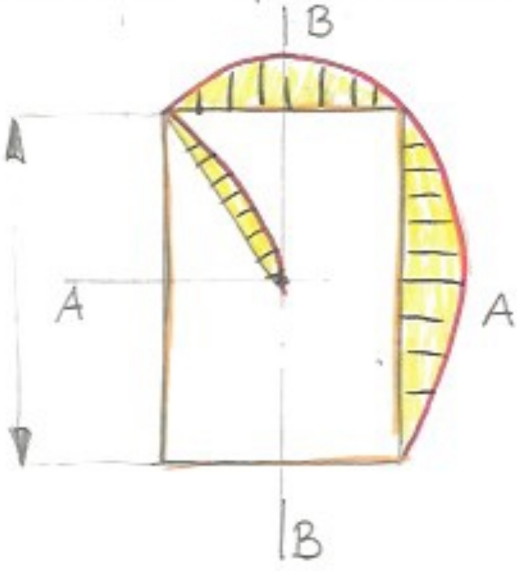
$$s_{21} = 58{,}7 \pm 56{,}06$$

$$s_2 = 114{,}76 \text{ mm}$$

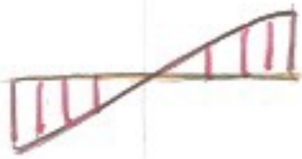
$$\underline{\underline{s_1 = 2{,}64 \text{ mm}}}$$

Hinweis auf Torsion nichtkreisförm. Querschnitte

- an den **Ecken** Torsionsspannung Ø (Null)
- nichtlineare Spannungsverteilung



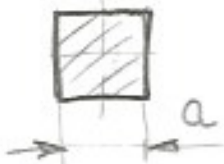
B-B



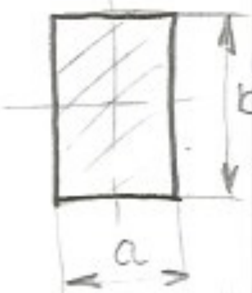
A-A

$$\tau_{tmax} = \frac{M_t}{W_t}$$

W_t nach Tabelle



$$W_t = 0{,}208\,a^3$$



$$W_t = \frac{c_1}{c_2}\,a^2 \cdot b$$

Tabelle A13

28.05.2012
17.05.2015

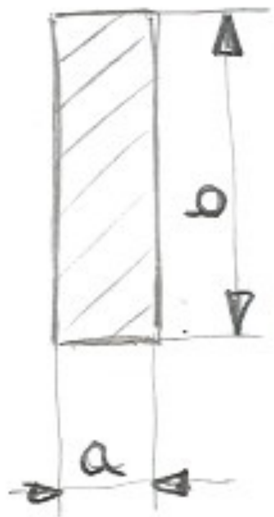
$$\boxed{\tau_{t\,max} \approx 3 \quad \frac{M_t}{a^2 \cdot b}}$$

Näherungsweise

$n = \frac{b}{a}$	$\frac{c_2}{c_1}$	$f\ \%$
> 10	$\frac{1}{0{,}33} = 3{,}03$	$\frac{3{,}03 - 3}{3} \mathrel{\hat{=}} \underline{\underline{1\%}}$
4	$\frac{0{,}990}{0{,}281} = 3{,}52$	$\frac{3{,}52 - 3}{3} \mathrel{\hat{=}} \underline{\underline{17{,}4\%}}$

wegen der Rechnerei!

Torsion schmaler Rechteckquerschnitte



$$\tau_{t\,max} = \frac{M_t}{W_t}$$

$$W_t = \frac{c_1}{c_2} a^2 \cdot b \quad ; \; I_t = c_1 \cdot a^3 \cdot b$$

$$\tau_{t\,max} = \frac{M_t \cdot c_2}{c_1 \cdot a^2 \cdot b} = \frac{c_2}{c_1} \cdot \frac{M_t}{a^2 \cdot b}$$

$$\boxed{\tau_{t\,max} \approx \frac{3 M_t}{a^2 \cdot b}}$$ in der Mitte der langen Seiten

$n = \frac{b}{a}$

$n = \frac{b}{a}$	$\frac{c_2}{c_1}$	Fehler
>10	3,03	1 %
10	3,2	6,6%
8	3,25	8,3%
6	3,34	11,3%
4	3,52	17,3%
3	3,71	23,6%
2	4,05	35%
1	4,78	59,3%

$c_1 = 0{,}333$
$c_2 = 1$

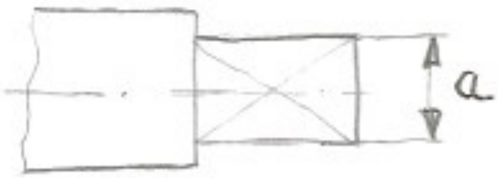
richtig:
$c_1 = 0{,}3$
$c_2 = 1$

$\frac{c_2}{c_1} = 0{,}333$

$F = \frac{0{,}33 - 0{,}3}{0{,}3}$

$\approx 1{,}1\%$

Bsp.: - Handwindekurbel



gege.: $\tau_t = 20$ MPa

$M_t = 70$ Nm

ges: a

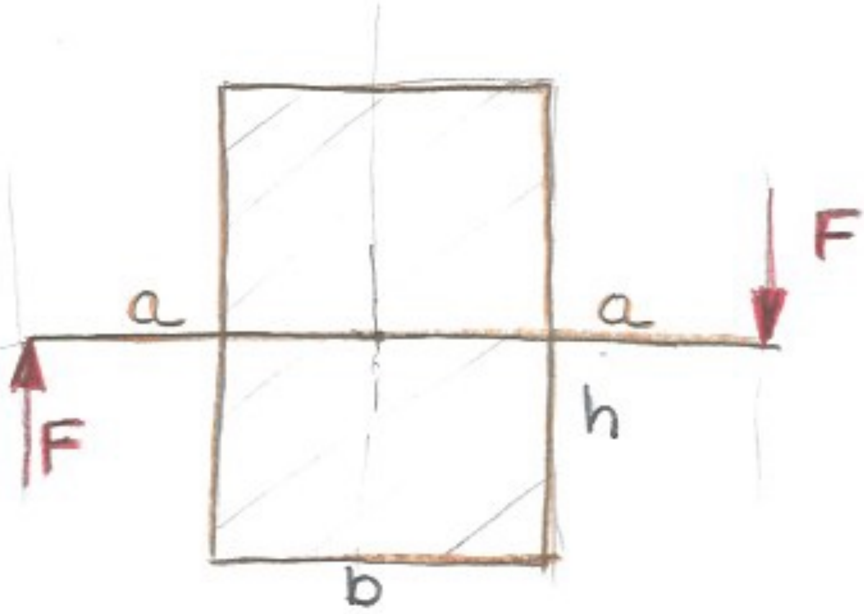
Lsg.:

$$\tau_t = \frac{M_t}{W_t} = \frac{M_t}{0{,}208\, a^3}$$

$$a = \sqrt[3]{\frac{M_t}{0{,}208 \cdot \tau_t}} = \sqrt[3]{\frac{70 \cdot 10^3}{0{,}208 \cdot 20}}$$

$a_{erf} = 25{,}62$ mm

$\underline{a = 27 \text{ mm}}$



geg.: $a = 50\,mm$
$b = 30\,mm$
$h = 50\,mm$
$\ell = 300\,mm$
$F = 12\,kN$

ges.: τ_t
$\varphi°$

Lösung:

$$\boxed{\tau_t = \frac{M_t}{W_t}}$$

$$\boxed{W_t = \frac{c_1}{c_2}\, a^2 \cdot b}$$

$$n = \frac{b}{a} = \frac{h}{b} = \frac{50}{30} = \frac{5}{3}$$

$a = b = 30\,mm$
$b = h = 50\,mm$

$$n = 1{,}6\bar{6}$$

$$c_1 = \frac{1}{3}\left(1 - \frac{0{,}630}{n} + \frac{0{,}052}{n^5}\right)$$

$$c_1 = 0{,}208$$

$$c_2 = 1 - \frac{0{,}65}{1+n^3} = 0{,}88$$

$$W_t = \frac{0{,}208}{0{,}88}\, 30^2 \cdot 50$$

$$W_t = 10{,}6\ cm^3$$

$$\underline{\underline{W_t = 10636{,}3\ mm^3}}$$

$$\tau_t = \frac{12 \cdot 10^3 \cdot 2 \cdot 50}{10636{,}3} = \underline{\underline{112{,}8\ N/mm^2}}$$

$$\varphi^\circ = 57{,}3^\circ\ \frac{M_t \cdot l}{G \cdot I_t}$$

$$I_t = c_1 \cdot n \cdot a^4$$

$$= 0{,}208 \cdot 1{,}66 \cdot 30^4$$

$$I_t = 279676{,}9\ mm^4$$

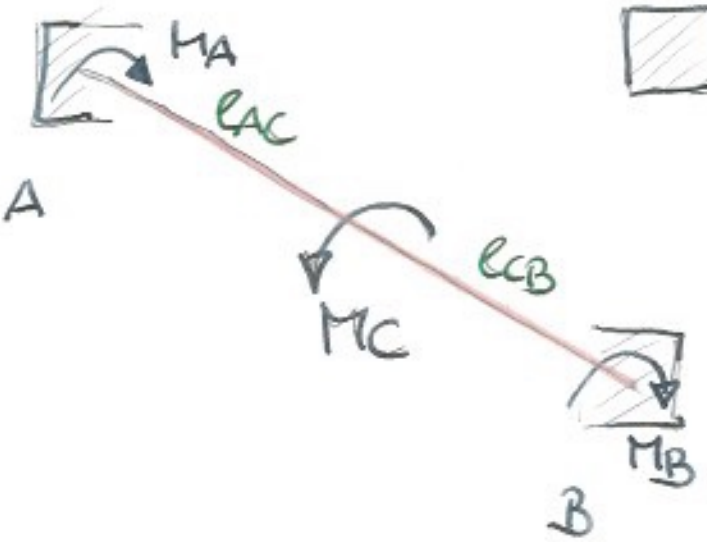
$$\varphi^\circ = 57{,}3^\circ\ \frac{12 \cdot 10^3 \cdot 2 \cdot 50 \cdot 300}{8 \cdot 10^4 \cdot 278679}$$

$$\varphi^\circ = 0{,}92^\circ \ \hat{=}\ 3\ ^\circ/m$$

27.05.2017

Hibbeler A.5.97

$$\varphi_t = \frac{7{,}1 \cdot M_t \cdot l}{a^4 G}$$



geg.:

$M_C = 80\,Nm$

$l_{AC} = 1m$

$l_{CB} = 1{,}5m$

$a = 50\,mm$

$G_{Al} = 26\,GPa \hat{=} 26 \cdot 10^3\,MPa$

ges.:

$M_A ; M_B$

φ_{AC}

Lösung:

$$M_A - M_C + M_B = 0$$

$$\varphi_{A-B} = 0$$

$$\varphi_{AC} - \varphi_{CB} = 0$$

$$\frac{M_A \cdot l_{AC}}{G \cdot I_t} - \frac{(M_C - M_A)\, l_{BC}}{G \cdot I_t} = \emptyset$$

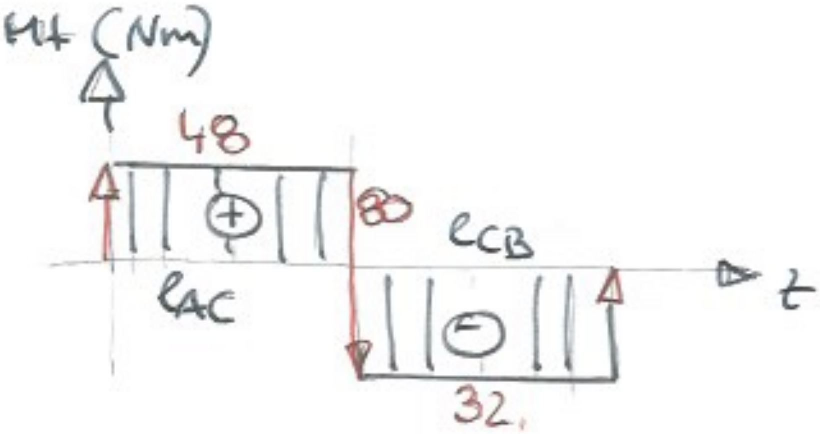
$$M_A \cdot l_{AC} = M_C \cdot l_{BC} - M_A \cdot l_{BC}$$

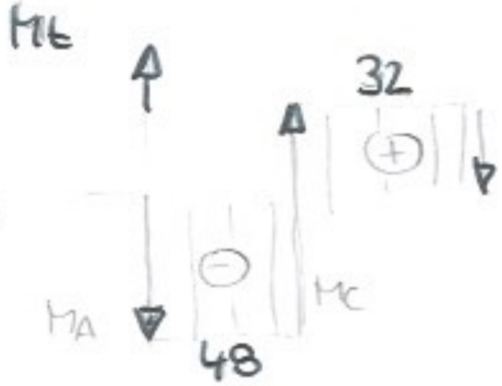
$$M_A = \frac{M_C \; l_{BC}}{l_{AC} + l_{BC}} = 80 \cdot \frac{1{,}5}{2{,}5}$$

$$\underline{\underline{M_A = 48\,Nm}}$$

$$M_B = M_C - M_A$$

$$M_B = 80 - 48 = \underline{\underline{32\ Nm}}$$





$$\varphi_{AC} = \frac{48 \cdot 10^3 \cdot 7{,}1 \cdot 1000 \cdot 180^\circ}{50^4 \cdot 26 \cdot 10^3 \cdot \pi}$$

$$\underline{\underline{\varphi_{AC} = 0{,}12^\circ}}$$