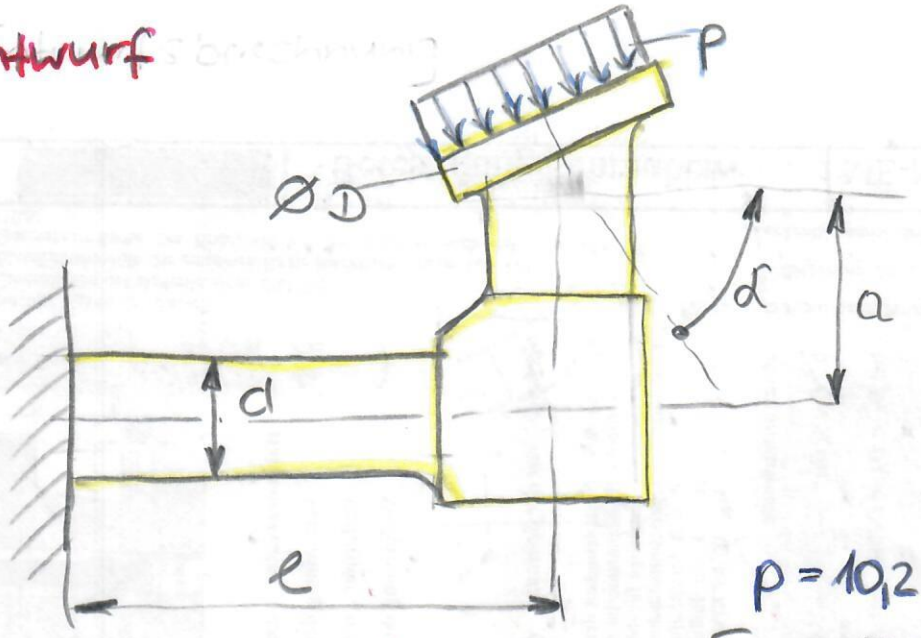
Entwurf

ges.: d



$p = 10{,}2$ bar

$\sigma_{zul} = 120\ N/mm^2$

$a = 500$ mm

$\ell = 1000$ mm

$D = 250$ mm

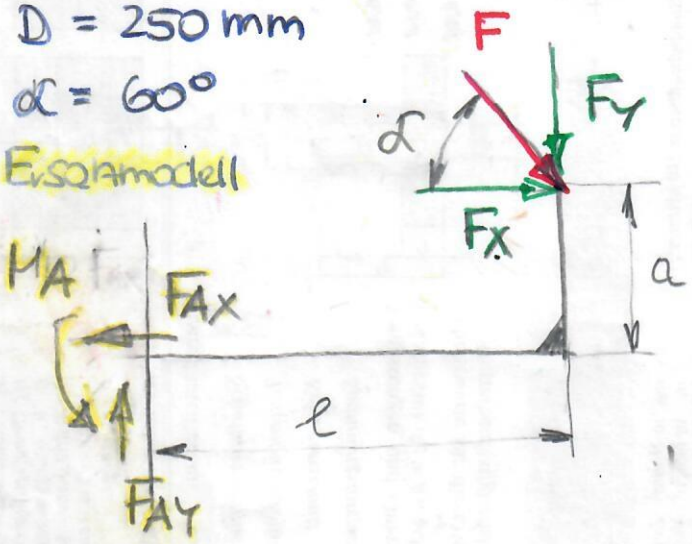
$\alpha = 60°$

$F = p \cdot A$

$= 1{,}02 \frac{N}{mm^2} \cdot \frac{\pi}{4} 250^2 mm^2$

$F = 50$ kN

Ersatzmodell



Auflager

$\rightarrow: F_{Ax} = F \cos\alpha$

$\uparrow: F_{Ay} = F \cdot \sin\alpha$

$\curvearrowleft M_A:$

$M_A - F\cos\alpha\, a - F \cdot \sin\alpha \cdot \ell = 0$

$M_A = M_b = F\cos\alpha\, a + F \cdot \sin\alpha \cdot \ell$

$= 50\ kN (\cos 60° \cdot 500 + \sin 60° \cdot 1000)$

$M_b = 5580$ kNcm

Regel: Der Entwurf wird **zunächst** mit reiner **Biegebeanspr.** durchgeführt, dann wird der **Querschnittskennwert** erhöht (oder nicht) u. dann **zusammengesetzte Beanspr.** nachgerechnet.

Entwurf:

$$\sigma_b = \frac{M_b}{W} \leq \sigma_{b\,zul} \qquad W = 0{,}1 \cdot d^3$$

$$\sigma_{b\,zul} = 120\,N/mm^2$$

$$M_b = 5580\,kNcm$$

$$d = \sqrt[3]{\frac{M_b}{0{,}1 \cdot \sigma_{b\,zul}}}$$

$$= \sqrt[3]{\frac{5580 \cdot 10^4}{0{,}1 \cdot 120}} = 16{,}7\,cm$$

$$d_{gew} = 17\,cm$$

Nachrechnung:

$$\sigma_{res} = + \frac{F \cdot \cos\alpha}{\frac{\pi}{4} d^2} + \frac{M_b}{0{,}1 \cdot d^3}$$

$$\sigma_{res} = \frac{50 \cdot 10^3 \cdot \cos 60^\circ}{\frac{\pi}{4} \cdot 170^2} + \frac{5580 \cdot 10^4}{0{,}1 \cdot 170^3} \quad (Zug)$$

$$\sigma_{res} = 1{,}1 + 113{,}5 \approx 115\,\frac{N}{mm^2} < 120\,\frac{N}{mm^2}$$

$$\sigma = \sigma_z + \sigma_b$$

$$\sigma = \frac{F \cdot \cos\alpha}{A_\odot} + \frac{M_b}{W_\odot}$$

$$\sigma = \frac{F \cdot \cos\alpha \cdot 4}{\pi d^2} + \frac{F(\cos\alpha \, a + \sin\alpha \cdot \ell) \cdot 32}{\pi d^3} \quad | \cdot d^3$$

$$\sigma d^3 - F \cdot \cos\alpha \cdot \frac{4}{\pi} d - F(\cos\alpha \cdot a + \sin\alpha \cdot \ell) \frac{32}{\pi} = 0$$

$$d^3 - \frac{4}{\pi} F \cdot \frac{\cos\alpha}{\sigma} d - \frac{F \cdot 32}{\pi \cdot \sigma} (\cos\alpha \cdot a + \sin\alpha \cdot \ell) = 0$$

$$d^3 - 265{,}258 \, d - 47365892 = 0$$

$$\underline{d_1 = 168{,}46 \text{ mm}}$$

d^3	0	$-265{,}258$	-47365892
168,46	168,46	$-28378{,}77$	4736002
168,461	168,46	$-28113{,}51$	0

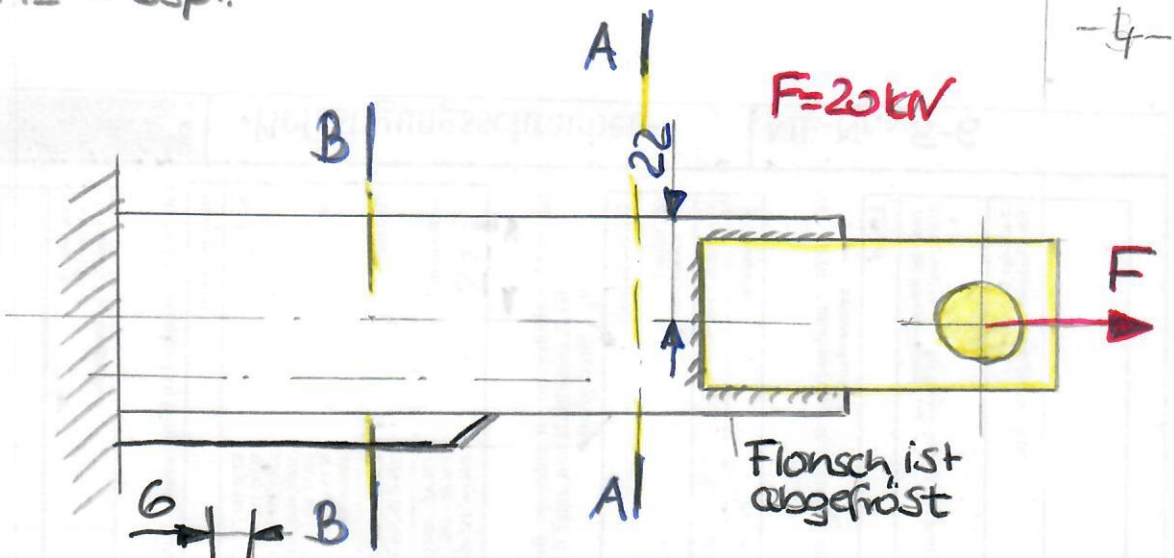
$$a^2 + 168{,}46\,a + 28113{,}51 = 0$$

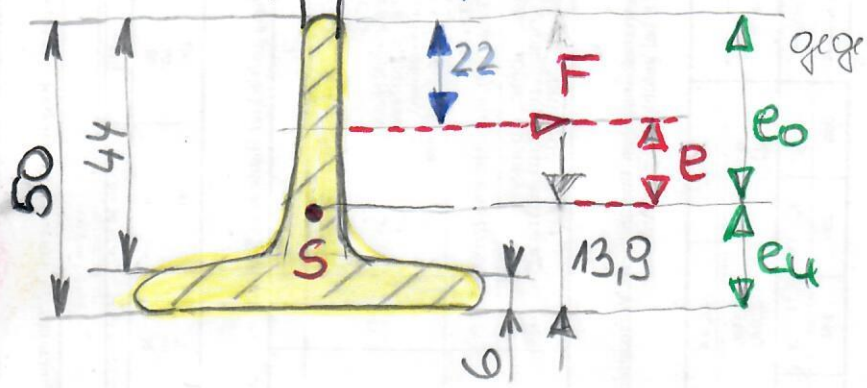
$$a_{2_3} = -84{,}23 \pm \sqrt{\text{n.e}}$$

↓

komplexe Zahl

TM ASME - Bsp.:





gegg.: T50-DIN EN 10055

$A = 5{,}66\,cm^2$

$W_x = 3{,}36\,cm^3$

$I_x = 12{,}1\,cm^4$

$e_o = 3{,}61\,cm$

$e_u = 1{,}39\,cm$

Schnitt A-A:

$$\sigma_z = \frac{F}{A_{\square}} = \frac{20 \cdot 10^3 N}{44 \cdot 6\,mm^2} = 75{,}75\,\frac{N}{mm^2}$$

Schnitt B-B: besser: $e_o - 22 = 1{,}41\,cm$

Hebelarm $e = 50 - 22 - 13{,}9 = 14{,}1\,mm = 1{,}41\,cm$

$$\sigma_{res} = \frac{F}{A_T} \pm \frac{F \cdot e}{I_x} \cdot e_o/e_u$$

Zug-oben

$$\sigma_{res} = +\frac{F}{A_T} + \frac{F \cdot e}{I_x} e_o = \frac{20 \cdot 10^3}{566} + \frac{20 \cdot 10^3 \cdot 14{,}1}{12{,}1 \cdot 10^4} \cdot 36{,}1$$

$$\sigma_{res} = 35{,}33 + 84{,}14 = 119{,}5\,N/mm^2$$

Druck-unten

$$\sigma_{res} = +\frac{F}{A_T} - \frac{F \cdot e}{I_x} \cdot e_u = \frac{20 \cdot 10^3}{566} + \frac{20 \cdot 10^3\ 14{,}1}{12{,}1 \cdot 10^4} \cdot 13{,}9$$

$$\sigma_{res} = 35{,}33 - 32{,}395 = 2{,}935\,N/mm^2$$

Prüfungsvorbereitung

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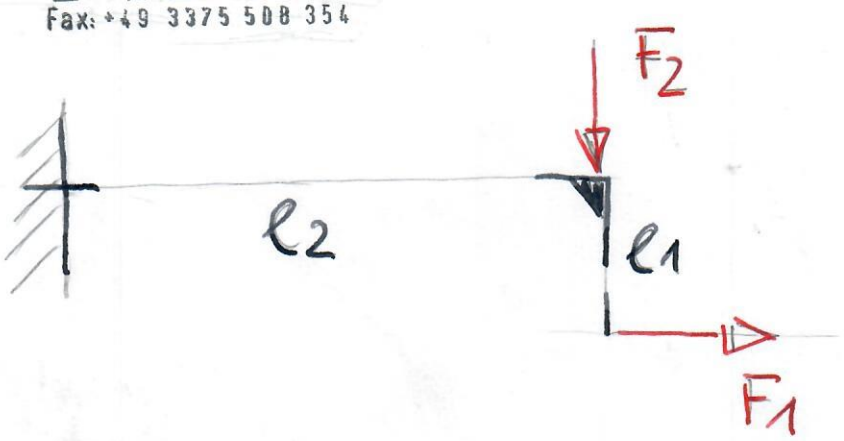
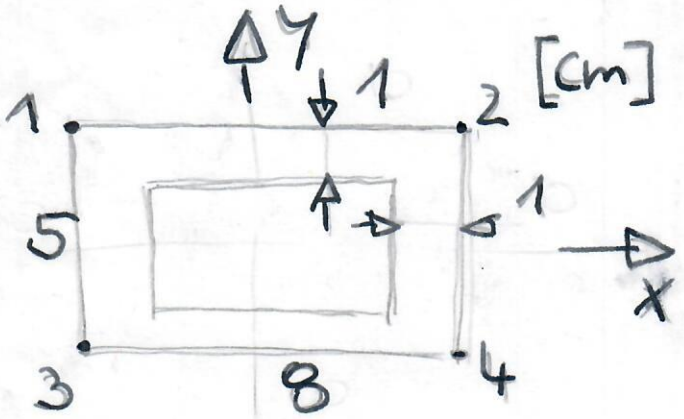
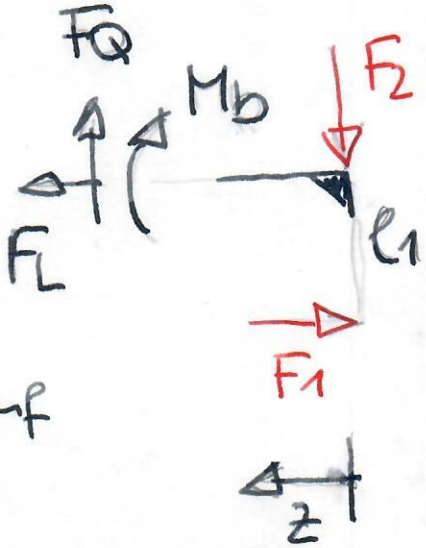


$F_1 = 40\,kN$ $\quad l_1 = 10\,cm$

$F_2 = 50\,kN$ $\quad l_2 = 20\,cm$





Wenn Momentenverlauf gefragt ist

$z(0 \ldots l_2)$

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$$M_b + \overline{F_2} \cdot z - \overline{F_1} \cdot \ell_1 = 0$$

$$M_b = \overline{F_1} \cdot \ell_1 - \overline{F_2} \cdot z =$$

$z = 20\,cm$

$$= 40 \cdot 10 - 50 \cdot 20$$

$$= -600\,kNcm$$

f. Schnittreaktionen

hier gehts los!

Einspannstelle:

$$M_x = -\overline{F_1} \cdot \ell_1 + \overline{F_2} \cdot \ell_2$$

$$M_x = 600\,kNcm$$

$$I_x = \frac{B \cdot H^3}{12} - \frac{b \cdot h^3}{12}$$

$$I_x = \frac{8 \cdot 5^3}{12} - \frac{6 \cdot 3^3}{12}$$

$$\underline{I_x = 69{,}83\,cm^4}$$

$$A = 5 \cdot 8 - 3 \cdot 6 = \underline{22\,cm^2}$$

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$$\sigma_{res} = \sigma_z \pm \sigma_b$$

$$\sigma_{res} = \frac{F_1}{A} \pm \frac{M_x}{I_x} \cdot y$$

$P_1(-4; 2{,}5)$
$P_2(4; 2{,}5)$
$P_3(-4; -2{,}5)$
$P_4(4; -2{,}5)$

$$\sigma_{res} = \frac{40}{22} + \frac{600}{69{,}83} y$$

$$\sigma_{res} = 1{,}81 \frac{kN}{cm^2} + 8{,}59 \frac{kNcm}{cm^4} \cdot y$$

$$\sigma_{1,2} = 1{,}81 + 8{,}59 \cdot 2{,}5 = 23{,}28 \frac{kN}{cm^2} \qquad \cdot 10^3 \cdot 10^{-2}$$

$$\sigma_{1,2} = 232{,}8 \frac{N}{mm^2} \approx \underline{\underline{233 \frac{N}{mm^2}}}$$

$$\sigma_{3,4} = 1{,}81 - 8{,}59 \cdot 2{,}5 = -19{,}665 \frac{N}{mm^2}$$

$$\sigma_{3,4} = \underline{\underline{-196{,}7 \frac{N}{mm^2}}}$$

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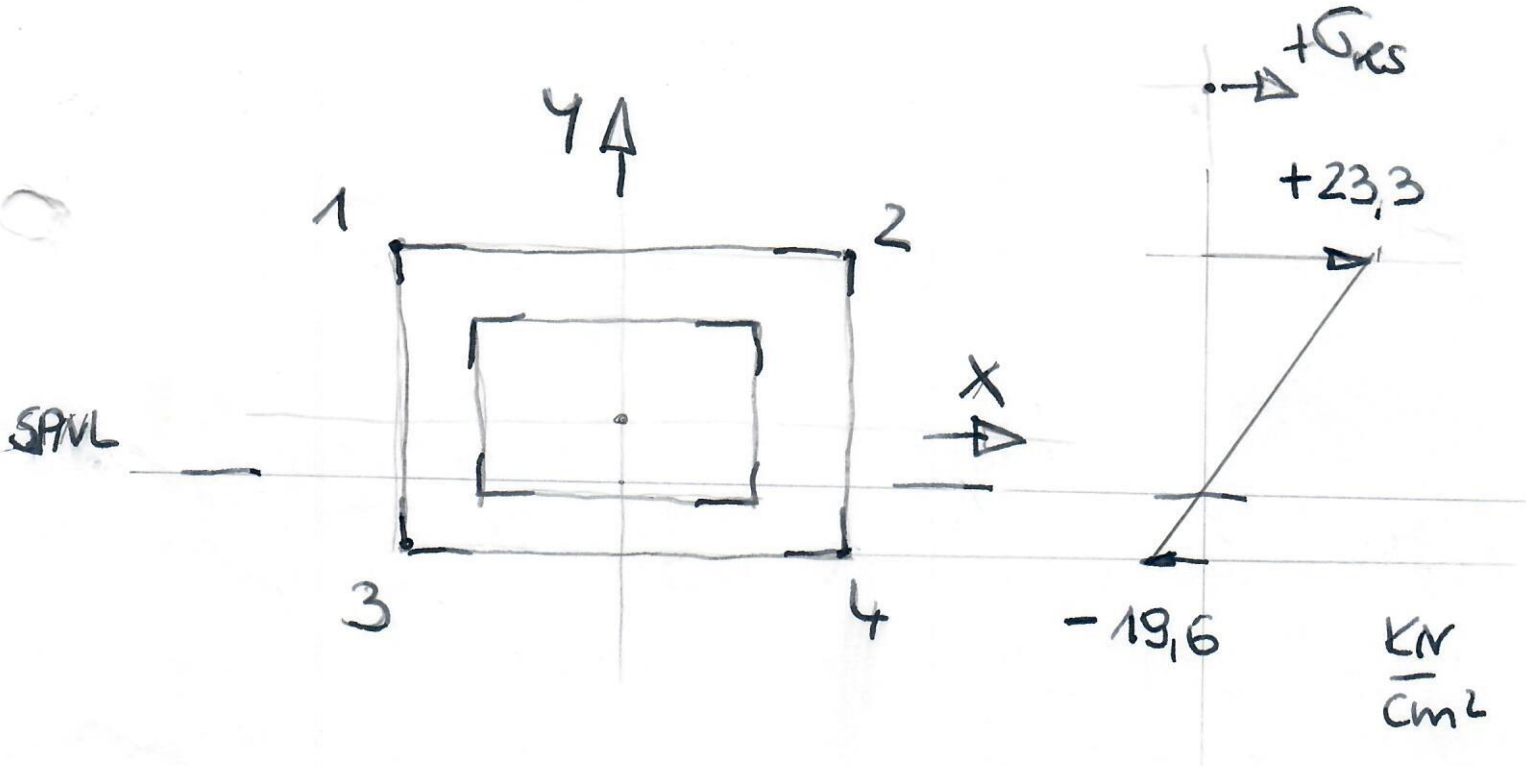
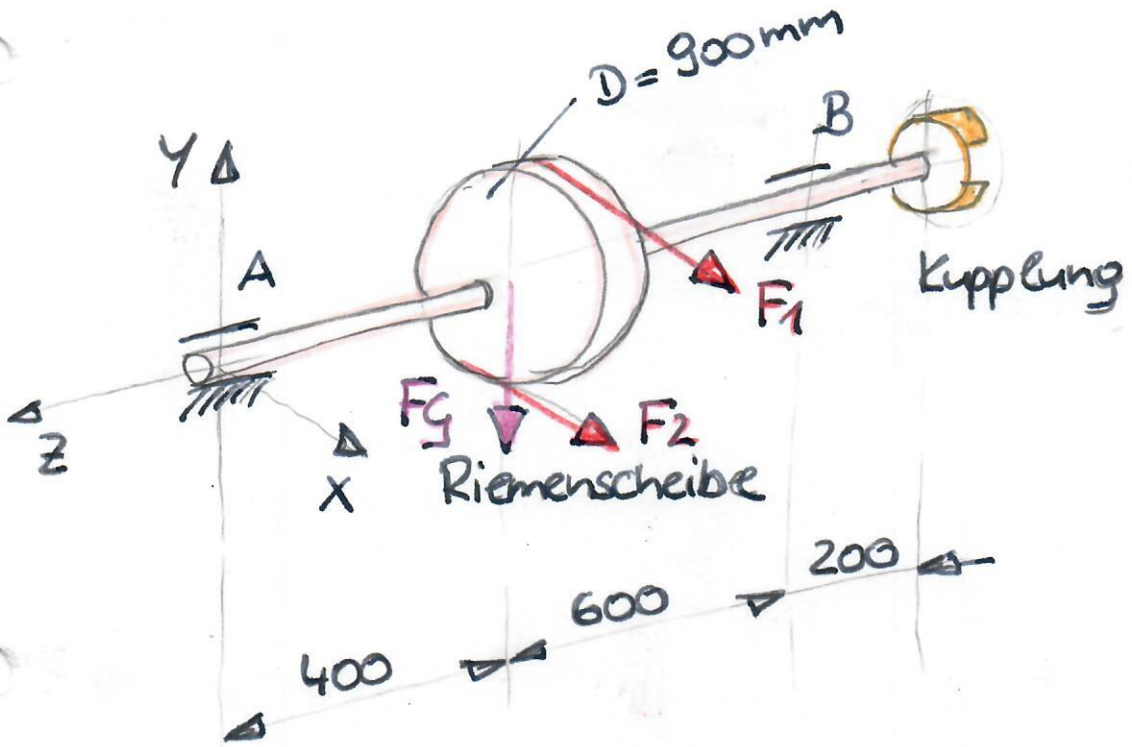
Sp NL:

$$0 = \sigma_{RS} = 1{,}81 + 8{,}59\ y \qquad \frac{kN}{cm^2}\ \frac{cm^3}{kN}$$

$$y = -0{,}21\ cm$$



Beispiel 2:



gege.: F_1 - Arbeitstrum = 3,9 kN

F_2 - Leertrum = 1,9 kN

F_G = 800 N

B: LF III

T: LF II

$\sigma_{bzul} = 105 \frac{N}{mm^2}$

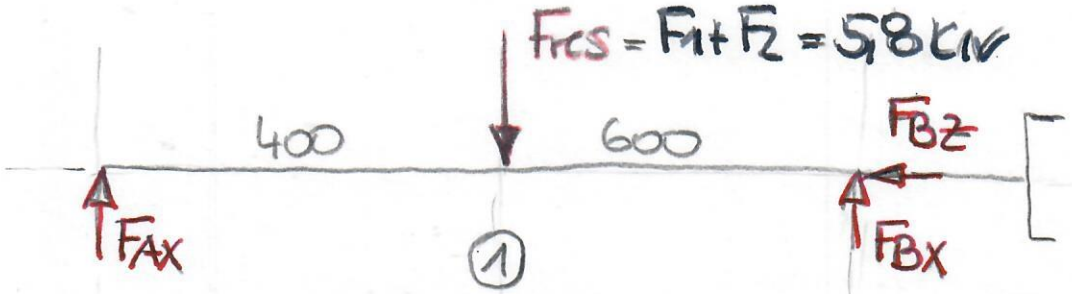
ges.: derf

$$d_{erf} = \sqrt[3]{\frac{32 \cdot M_v}{\pi \cdot \sigma_{bzul}}}$$

$$M_v = \sqrt{M_{b_{res}}^2 + 0{,}4 \cdot M_t^2}$$

Mres - bestimmen

X-Y-Ebene



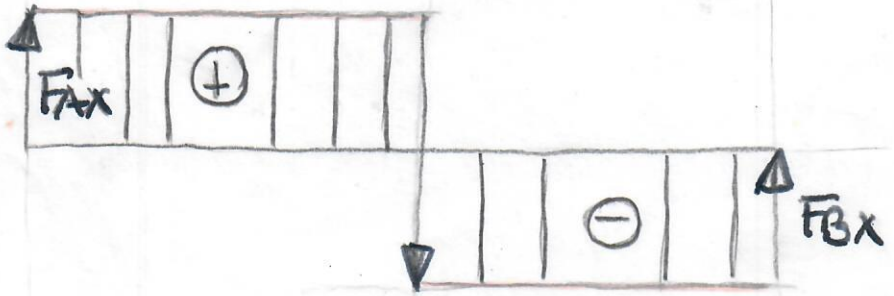
$$F_{AX} = \frac{600}{1000} \cdot F_{res}$$

$$F_{AX} = \frac{6}{10} 5{,}8$$

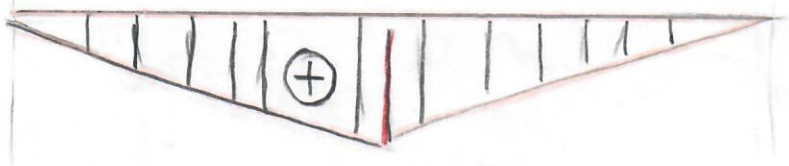
$$F_{AX} = 3{,}48\ kN$$

$$F_{BX} = \frac{4}{10} \cdot 5{,}8$$

$$F_{BX} = 2{,}32\ kN$$



$$M_{b_{(1)}} = F_{AX} \cdot 400$$
$$= F_{BX} \cdot 600$$

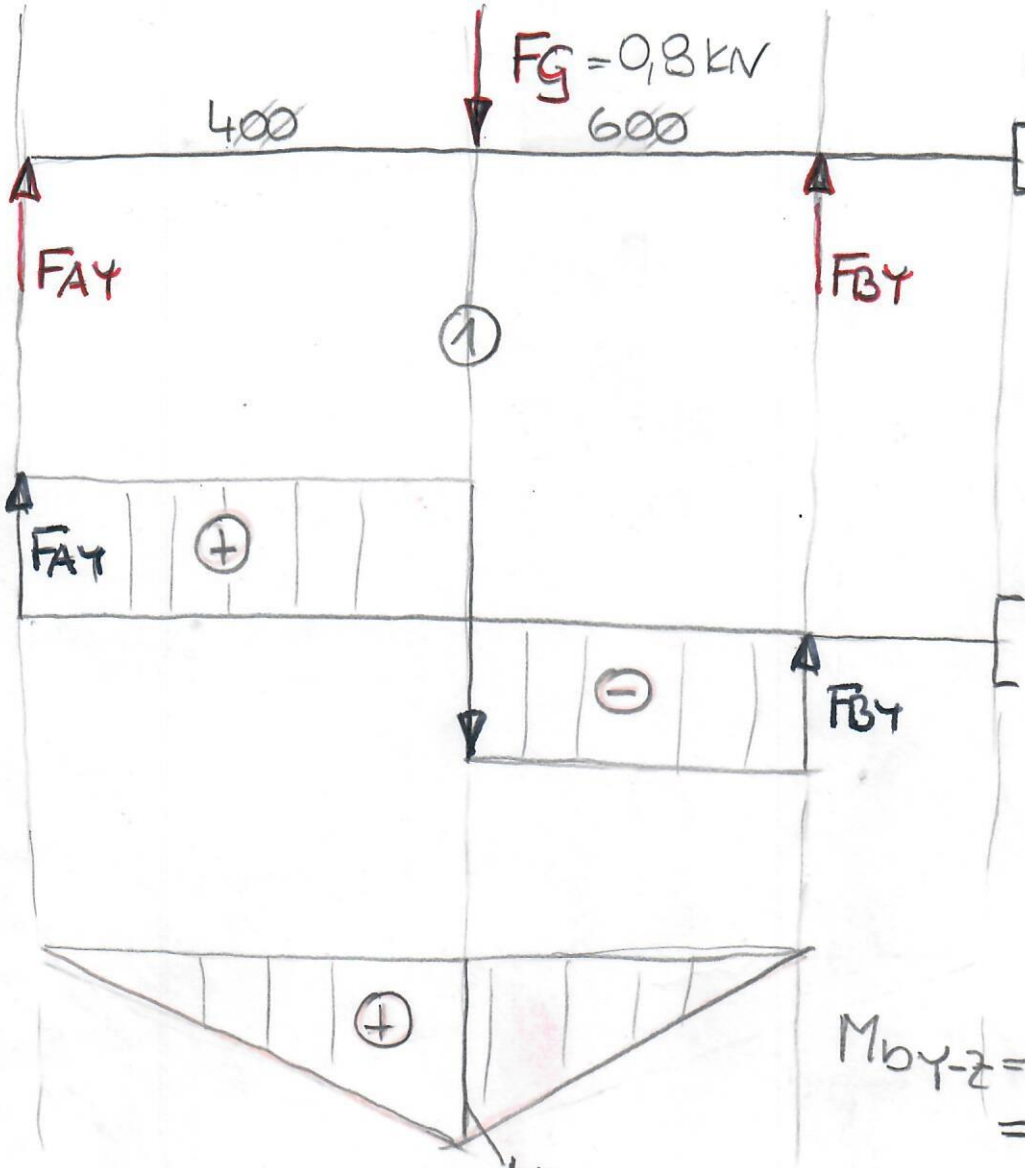


$$M_{b_{(1)}} = 3{,}48 \cdot 0{,}4$$

$$M_{b_{(1)}} = 1{,}39\ kNm$$

$M_{b_{(1)}\ x\text{-}y} = 1{,}39\ kNm$

Y-Z-Ebene



$F_{AY} = \frac{6}{10} F_G$

$F_{AY} = 948\,kN$

$F_{BY} = \frac{4}{10} F_G$

$F_{BY} = 932\,kN$

$M_{bY-Z} = F_{AY} \cdot 0{,}4$
$= F_{BY} \cdot 0{,}6$

$M_{b①\,Y-Z} = 948 \cdot 0{,}4$
$= 0{,}192\,kNm$

$$M_{bKS①} = \sqrt{M_{bX-Y}^2 + M_{bY-Z}^2}$$

$$M_{bKS①} = \sqrt{1{,}39^2 + 0{,}192^2} = 1{,}4\,kNm$$

$$d_{erf} = \sqrt[3]{\frac{32 \cdot M_V}{\pi \cdot \sigma_{bzul}}}$$

$$M_V = \sqrt{M_{bres}^2 + 0{,}4\, M_t^2}$$

~~$M_t = M_z = \emptyset$~~

$$M_z + F_2 \cdot R - F_1 \cdot R = \emptyset$$

$$M_z = R(F_1 - F_2) = 0{,}45\,(3{,}9 - 1{,}9)$$

$$M_z = M_t = \underline{0{,}9}\ KNm$$

$$M_V = \sqrt{1{,}4^2 + 0{,}4 \cdot 0{,}9^2}$$

$$M_V = 1{,}51\ KNm$$

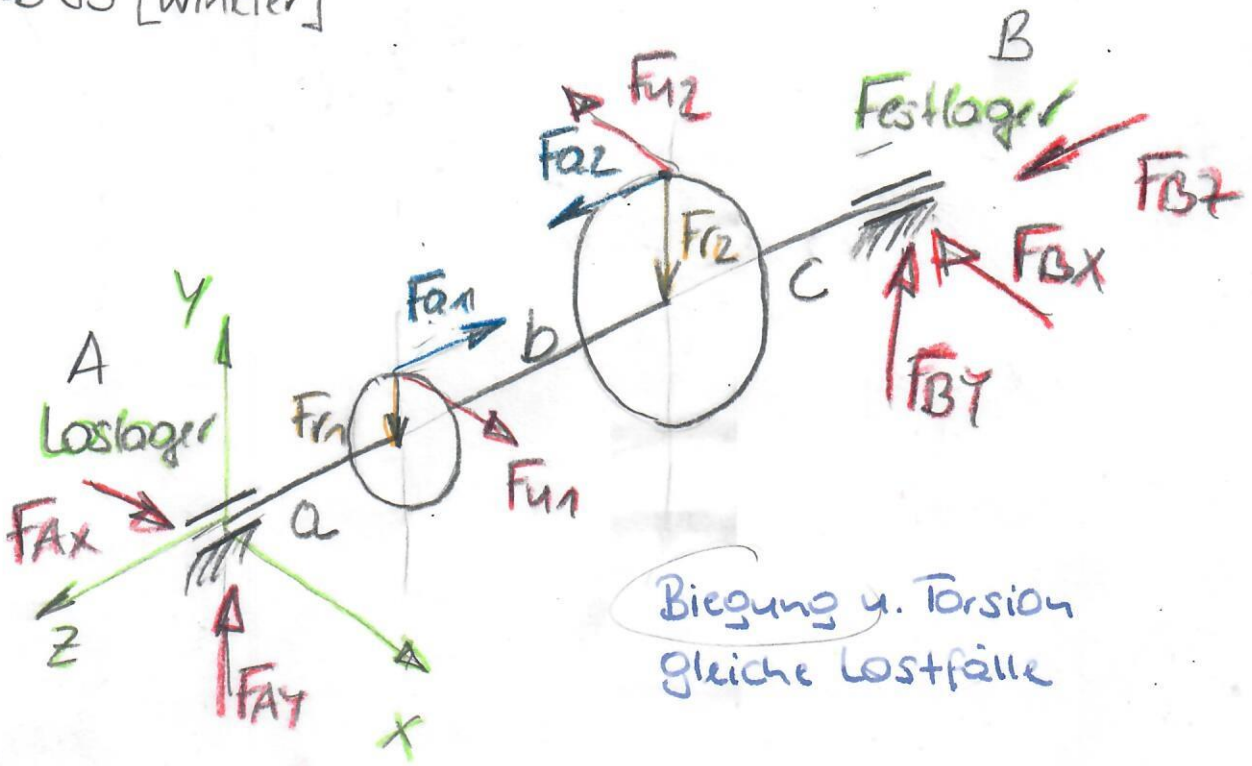
$$d_{erf} = \sqrt[3]{\frac{32 \cdot 1{,}51 \cdot 10^6}{\pi \cdot 105}}$$

$$d_{erf} = 52{,}7\ mm$$

Passfeder $d_1 \begin{matrix} 50 \\ 58 \end{matrix}$ $t_1 = 6\ mm$

$$d_{gew} = 60\ mm$$

LB 95 [Winkler]



Biegung u. Torsion
gleiche Lastfälle

gegeben: $\sigma_{bzul} = 80\,MPa$ gesucht: d

$F_{u1} = 15{,}25\,kN$	$F_{u2} = 5{,}46\,kN$
$F_{a1} = 4{,}13\,kN$	$F_{a2} = 2{,}45\,kN$
$F_{r1} = 5{,}6\,kN$	$F_{r2} = 1{,}93\,kN$
$r_{01} = 50{,}5\,mm$	$r_{02} = 141\,mm$

$a = c = 60\,mm$
$b = 160\,mm$

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Lösung:

$\uparrow: F_{Ay} - F_{r1} - F_{r2} + F_{By} = 0$

$\rightarrow: F_{a1} - F_{a2} - F_{Bz} = 0$

$\searrow: F_{u1} + F_{Ax} - F_{Bx} - F_{u2} = 0$

$\circlearrowleft F_{Ay}: -F_{u1} \cdot a + F_{u2}(a+b) + F_{Bx}(a+b+c) = 0$

$$F_{Bx} = \frac{F_{u1} \cdot a - F_{u2}(a+b)}{a+b+c} = \frac{15{,}25 \cdot 60 - 5{,}46 \cdot 220}{280}$$

$$F_{Bx} = -1{,}02\,kN$$

↶ F_{BX}:

$$F_{Q2} \cdot r_{\varnothing 2} + F_{r2} \cdot c - F_{Q1} \cdot r_{\varnothing 1} + F_{r1}(b+c) - F_{AY}(a+b+c) = 0$$

$$F_{AY} = \frac{F_{Q2} \cdot r_{\varnothing 2} + F_{r2} \cdot c - F_{Q1} \cdot r_{\varnothing 1} + F_{r1}(b+c)}{a+b+c}$$

$$F_{AY} = \frac{2{,}45 \cdot 141 + 1{,}93 \cdot 60 - 4{,}13 \cdot 50{,}5 + 5{,}6 \cdot 220}{280}$$

$$\underline{\underline{F_{AY} = 5{,}3\ kN}}$$

↶ F_{BY}: $+F_{Ax}(a+b+c) + F_{u1} \cdot (b+c) - F_{u2} \cdot c = \varnothing$

$$F_{Ax} = \frac{F_{u2} \cdot c - F_{u1}(b+c)}{a+b+c}$$

$$F_{Ax} = \frac{5{,}46 \cdot 60 - 15{,}25 \cdot 220}{280}$$

$$\underline{\underline{F_{Ax} = -10{,}81\ kN}}$$

→ F_{Bz}: $-F_{Bz} - F_{Q2} + F_{Q1} = \varnothing$

$$F_{Bz} = F_{Q1} - F_{Q2} = 4{,}13 - 2{,}45$$

$$\underline{\underline{F_{Bz} = 1{,}68\ kN}}$$

$\sum F_{AX}: \quad F_{BY}(a+b+c) - F_{r2}(b+a) + F_{a2} \cdot r_{\varnothing 2} - F_{r1} \cdot a - F_{a1} \cdot r_{\varnothing 1} = 0$

$$F_{BY} = \frac{F_{r2}(b+a) - F_{a2} \cdot r_{\varnothing 2} + F_{r1} \cdot a + F_{a1} \cdot r_{\varnothing 1}}{a+b+c}$$

$$F_{BY} = \frac{1,93 \cdot 220 \cdot 2,45 \cdot 141 + 5,6 \cdot 60 + 4,13 \cdot 50,5}{280}$$

$$\underline{\underline{F_{BY} = 2,23 \text{ kN}}}$$

oder:

$$F_{BY} = -F_{AY} + F_{r1} + F_{r2}$$

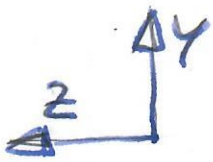
$$F_{BY} = -5,3 + 5,6 + 1,93$$

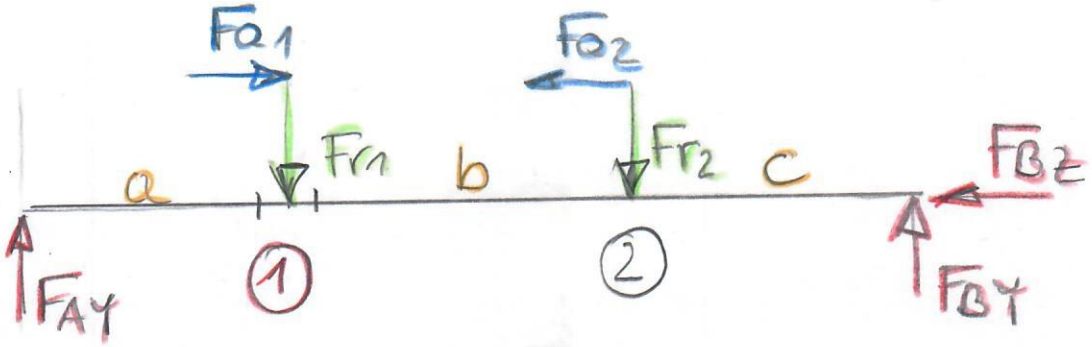
$$\underline{\underline{F_{BY} = 2,23 \text{ kN}}}$$

$$F_A = \sqrt{F_{AX}^2 + F_{AY}^2} = \sqrt{1,981^2 + 5,3^2} = 12 \text{ kN}$$

$$F_B = \sqrt{F_{BX}^2 + F_{BY}^2} = \sqrt{1,02^2 + 2,23^2} = 2,45 \text{ kN}$$

Ebene Y-Z





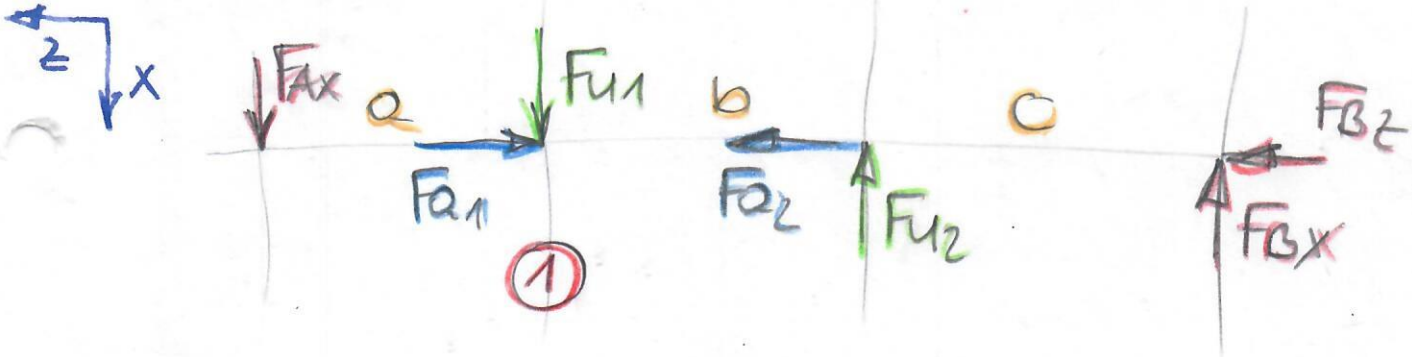
Stelle 1: $M_{b(Y-Z)①.1} = F_{AY} \cdot a = 5{,}3 \cdot 60 = 318\ kNmm$
$= 318\ Nm$

Stelle 2 $M_{b(Y-Z)①.2} = F_{AY} \cdot a + F_{Q1} \cdot r_{Ø1}$

$= 318 + 4{,}13 \cdot 50{,}5$

$M_{b(Y-Z)①.2} = \underline{526{,}56\ Nm}$

Ebene Z-X



$M_{b(X-Z)①} = -F_{AX} \cdot a$

$= 1091 \cdot 60 = 648{,}6\ Nm$

$M_{bres} = \sqrt{526{,}56^2 + 648{,}6^2} = \underline{836\ Nm}$

Stelle 2: $M_{bres_2} = \sqrt{478{,}25^2 + 61{,}2^2} = 483{,}14\ Nm$

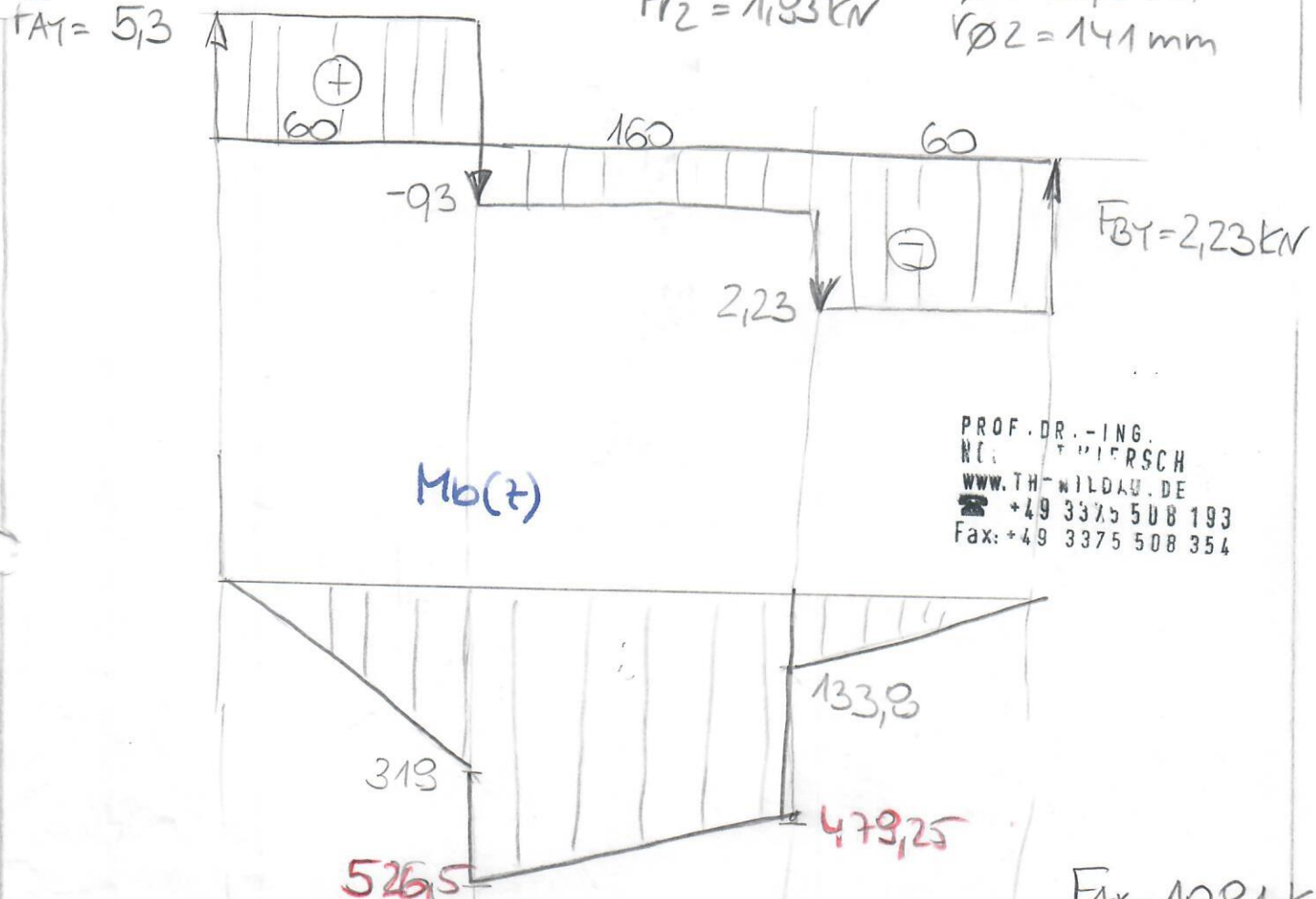
Ebene Y-z

$F_{AY} = 5{,}3\,kN$
$F_{BY} = 2{,}23\,kN$
$F_{r1} = 5{,}6\,kN$
$F_{r2} = 1{,}93\,kN$

$F_{Q1} = 4{,}13\,kN$
$F_{Q2} = 2{,}45\,kN$

$r_{\varnothing 1} = 595\,mm$
$r_{\varnothing 2} = 141\,mm$

$F_Q(z)$



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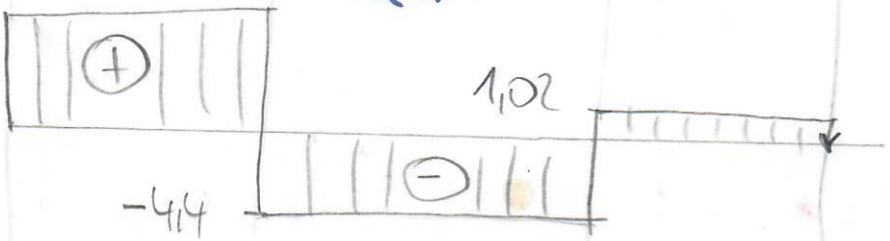
$M_b(z)$

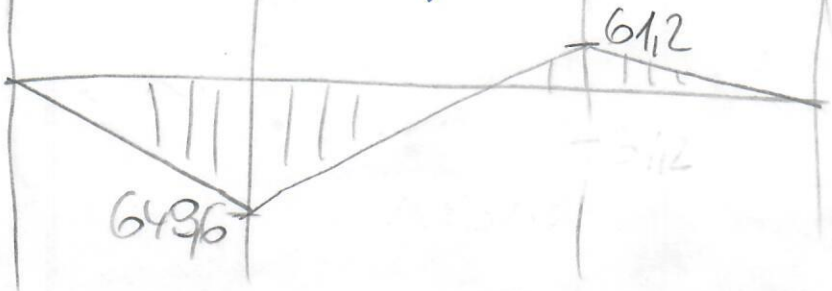
$F_{Ax} = 10{,}81\,kN$
$F_{Bx} = 1{,}02\,kN$
$F_{u1} = 15{,}25\,kN$
$F_{u2} = 5{,}46\,kN$

Ebene z-x

$F_Q(z)$



$M_b(z)$



Torsionsmoment:

$T = F_{U1} \cdot r_{Ø1} = 15{,}25 \cdot 50{,}5$

$T = 770\ Nm \quad (770{,}125)$

oder

$T = F_{U2} \cdot r_{Ø2} = 5{,}46 \cdot 141$

$T = 770\ Nm$

Vergleichsmoment: (Stelle 1)

$$M_V = \sqrt{M_{bres}^2 + \frac{3}{4} M_4^2}$$

B, T = LF III
§EH

$$M_V = \sqrt{836^2 + \frac{3}{4} 770^2}$$

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$M_V = 1069{,}37\ Nm$

$$d_{erf} = \sqrt[3]{\frac{M_V \cdot 32}{\pi \cdot \sigma_{dzul}}} = \sqrt[3]{\frac{1069{,}37 \cdot 10^3 \cdot 32}{\pi \cdot 80}}$$

$d_{erf} = 51{,}44\ mm$

$\underline{d_{gew} = 55\ mm}$